

Wage Adjustment: A Network Approach ^{*}

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Abstract

We study the role of strategic interactions between firms in wage setting and their implications for aggregate wage adjustment. Building on recent developments in the estimation of peer effects and utilizing granular administrative employee-employer data from Colombia, we identify the strategic interactions through a network-based approach. Our findings reveal that external shocks significantly influence firms' wage adjustments, even when firms are not directly exposed to these shocks. First, firms adjust their wages by approximately 2% in response to a 10% change in the wages of their competitors, controlling for external shocks. Second, firms respond to external shocks faced by their competitors, lowering wages by 10% in response to a 10% decline in the value added by their competing firms. The strategic interactions contribute to sluggish aggregate wage adjustment following negative external shocks. Our firm-to-firm network analysis shows the crucial role of larger and more productive firms in the transmission of shocks to the economy. External shocks first hit the above firms and propagate to the rest of the economy. To rationalize our empirical findings and conduct counterfactual analysis, we develop a general equilibrium model with rich firm and worker heterogeneity featuring strategic interactions between firms in wage setting.

Keywords: Strategic interactions in wage setting, sluggish wage adjustment, networks.

JEL: D22, E24, E32, J31, J63, J64, L25

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Introduction

Dating back to Keynes (1936), wage rigidity has been a central topic in macroeconomics. It is widely believed that wage rigidity is a key factor in the transmission of aggregate shocks to the broader economy. Specifically, aggregate shocks cause fluctuations in economic activity and employment due to the lack of wage flexibility. Given its policy relevance, significant attention has been paid by academics to incorporate wage (and price) rigidity into theoretical models, with two leading approaches emerging. The first approach introduces rigidities through nominal frictions.¹ That is, economic agents limit wage adjustment due to the costly nature in the nominal terms. The second approach focuses on strategic interactions among agents in wage setting, where coordination between firms may generate real rigidities, as firms avoid deviating too far from their competitors. While much progress has been made on the theoretical side, little do we know empirically about forces *behind* wage rigidity and its sluggish adjustment.

This paper contributes empirically to the existing literature by identifying one of the forces *behind* wage rigidity. Specifically, it measures the role of strategic coordination in wage setting, which is considered one of the main forces of *real* rigidities. The paper examines the strength of strategic interactions in wage setting and their implications for aggregate wage adjustment.² Strategic interaction arises from the competing nature of firms for workers by setting wages. In an environment where some, but not necessarily all, firms are exposed to negative external shocks, aggregate wage adjustment could exhibit sluggish behavior. This sluggishness can largely be explained by strategic interactions between firms. First, firms that are not necessarily exposed to external shocks could limit their wage adjustment due to direct exposure of firm-competitors to external shocks – referred throughout the paper as exogenous peer effects. Second, the exogenous peer effects can be amplified by endogenous peer effects. That is, firms also adjust their wages in response to their competitors' wage adjustments.

This paper adopts a novel approach and data to identify strategic interactions between firms in wage setting. First, we leverage recent development in the estimation of peer effects, which address two major econometric challenges in identifying strategic interactions. On the one hand, wage adjustment is a simultaneous process, meaning that a simple OLS procedure would produce biased

¹See, for example, Taylor (1980), Calvo (1983), Erceg et al. (2000), and Schmitt-Grohe & Uribe (2016).

²Throughout the paper, the notations of “strategic interactions”, “strategic complementarities” and “peer effects” are used interchangeably.

estimates. To overcome this, we use external shocks to firm-competitors and the wage adjustment of competitors' competitors as instrumental variables for the wage adjustment of direct competitors. On the other hand, by using worker-level wage data, we difference out unobservable worker wage pay components. Failing to do so would result in a mechanical correlation between wages. Second, we use administrative employee-employer data, which enables us to observe potential firm competitors. By using worker flows as a proxy for competition, we directly construct firm-competitor relationships, rather than regressing change in wages against changes in average "labor market" wages. We identify firms as competitors if they "poach" workers from each other. Additionally, we estimate worker flows between firms as a network formation - allowing us to account for the endogenous nature of the network formation.

We use the Colombian economy as the setting for experiment. First, the Colombian economy experienced a significant exchange rate depreciation in 2014-2015. As an economy heavily reliant on the export of petroleum products, coal, and basic metals; the Colombian peso is influenced by the international prices of these natural resources. Figure B.1 demonstrates the appreciation of the Colombian peso in the early 2000s with a relatively stable exchange rate in 2010-2014, and then sharp depreciation in late 2014 and 2015. Additionally, both nominal and real exchange rates surged, indicating that the exchange rate depreciation had little pass-through to domestic prices. Second, the economy is comprised of various industries that have differing levels of exposure to external shocks. As our leading measure of exposure to external shocks, we use the interaction of the 2-digit industry share of foreign intermediate inputs and exchange rate depreciation. Tables A.3 and A.4 illustrate the industry coverage across the entire economy. Third, we use administrative employee-employer data that allows us to observe the number of days worked; therefore, enabling us to base our analysis on intensive pay measures rather than annual wages.

Our main empirical finding reveals that firms strategically coordinate with their competitors when setting wages. The strategic interactions have endogenous and exogenous components. First, firms respond directly to their competitors' wage adjustment, independent of the average wage change in a given labor market. Specifically, a 10 percentage point increase in the average wage of a firm's competitors leads to approximately 1 percentage point adjustment in the firm's wage. We refer to this component as the endogenous peer effect. Second, firms also react indirectly to external shocks faced by their competitors. For instance, a 10 percentage point increase in the value of intermediate inputs

of a firm’s competitors results a 6 percentage point reduction in the firm’s wage growth. We label this component as the exogenous peer effects.

To account for the endogenous nature of worker flows between firms, we estimate network formation at the firm level. Three key empirical findings emerge, offering additional insights into the forces driving aggregate wage adjustment. First, network formation demonstrates a strong degree of homophily. Firms within the same 1-digit or 2-digit industries are more likely to be connected by worker flows than firms from different industries. Additionally, networks are more likely to form between firms employing workers with similar observable and unobservable characteristics, such as age, gender, and ability. Second, the sizes of both the “sender” and “receiver” firms increase the average flow of workers between them. This aligns with the idea that larger firms are more likely to hire and fire workers on average. However, the size of the “receiver” firm has a larger effect, indicating that workers are more likely to move from smaller firms toward larger ones. This finding is in line with Haltiwanger, Hyatt, Kahn & McEntarfer (2018), who shows that workers progress up the size job ladders. Third, consistent with Haltiwanger, Hyatt & McEntarfer (2018), we find that workers also progress up the wage job ladder. Firms lower on the wage job ladder compete with firms higher up, in the line with the standard wage posting model *à la* Burdett & Mortensen (1998).

Our estimates demonstrate that strategic interactions significantly dampen aggregate wage adjustment in the short and medium runs. This dampening effect is quantified by taking into account the estimates of the endogenous and exogenous peer effects, weighted by firms’ contribution to aggregate wages with their employment size. Due to the strategic interactions in wage setting and network effects, aggregate wages in labor markets affected by external shocks could be approximately 2 percentage points lower over a two-year horizon. That is, even if firms are not directly affected by external shocks they tend to delay wage adjustment, exacerbating the sluggish aggregate wage adjustment.

To illustrate the policy implications, we develop a New-Keynesian small open economy model featuring oligopsonistic labor markets subject to both nominal and real frictions. The oligopsonistic labor markets is subject to nominal and real frictions. First, firms reset their labor contracts according to Calvo (1983), allowing us to capture the reality where labor contracts are renegotiated over a set horizon. Second, even when firms can reset labor contracts periodically, they still strategically interact with their competitors in the labor market. This strategic interaction introduces real rigidity. We model the strategic interactions following Kimball (1995). Our model demonstrates that when

monetary policy follows a standard Taylor rule aimed at stabilizing inflation, external shocks - such as commodity or capital outflow shocks - lead to exchange rate depreciate and create sluggish wage adjustment. Moreover, the strategic interactions between firms amplify these shocks, keeping real wages lower for a prolonged period, which, in turn, contributes to higher unemployment. In contrast, a monetary policy focused on stabilizing the nominal exchange rate helps sustain real wages and improves overall economic conditions.

This paper contributes to three strands of literature. First, it adds to the literature on strategic interactions in wage and price setting. The study most closely related to ours is Amiti et al. (2019), which identifies strategic interactions between firms in price setting and demonstrates their implications for exchange rate pass-through to prices. Unlike their approach, this paper uses a network-based method to identify strategic interactions, defining “direct” firm competitors rather than relying on market averages. The first empirical study to estimate monopsonistic power and strategic interactions in labor markets is Staiger et al. (2010), which examines an exogenous wage change at Department of Veterans Affairs hospitals and how it affects wages at nearby hospitals. They find evidence of strategic interactions and monopsonistic power. In contrast, our paper estimates strategic interactions across the entire economy and examines their implications for aggregate wage adjustment. Chan et al. (2024) investigate strategic interactions in wage setting and their effects on spatial inequality, while Droste (2024) estimates strategic interactions in posted wages in the U.S., leveraging changes in postings by national wage setters.

Second, this paper contributes to the strand of literature that studies the effects of exchange rate shocks on investment, employment, and productivity. Verhoogen (2008) utilizes the 1994 Mexican peso crisis as a natural experiment and demonstrates that export-oriented firms upgraded the quality of their products, which in turn led to higher employment and wage bills. Using industry-level data, Campa & Goldberg (1995) shows that exchange rate shocks significantly affect investment, with low-markup industries experiencing higher pass-through, while high-markup industries exhibit lower pass-through of the shocks to investment. Nucci & Pozzolo (2001) extends this analysis using firm-level data from manufacturing industries in Italy, confirming these findings. Ekholm et al. (2012), leveraging the Norwegian krone appreciation as a quasi-natural experiment, shows that while employment declined, net exporters experienced an increase in productivity due to heightened competition from foreign firms. Barbiero (2021) uses French firm-level microdata to explore the effects

of exchange rate shocks on cash flows, investment, and salaries using dollar invoicing shares as an instrumental variable. The most closely related paper to ours is Blanco et al. (2020), which demonstrates a decline in labor income inequality following the Argentinian peso devaluation, driven by low labor income mobility and a lack of union coverage among high-income workers. In contrast, our paper emphasizes the role of the strategic interactions between firms in wage setting and their implications for wage adjustment.

Third, this paper relates to the literature on wage cyclicality over the business cycle. The seminal work by Bils (1985) highlights the importance of controlling for unobservable worker characteristics to mitigate compositional bias in measuring the cyclicality of aggregate wages. He demonstrates that wages for both job-stayers and job-movers are procyclical, with a higher degree of procyclicality for job-movers; this finding further emphasized by Haefke et al. (2013). Kudlyak (2014) constructs an empirical measure of the user cost of labor and shows that it is substantially more procyclical than either average wages or wages of newly hired workers. However, more recent studies, such as Gertler et al. (2020), Hazell & Taska (2020), and Grigsby et al. (2021), suggest that wages are weakly procyclical. The discrepancy arises from the challenge of accounting for changes in job composition. Similarly, Haltiwanger, Hyatt, Kahn & McEntarfer (2018) underscores that shifts in job composition over the business cycle substantially contribute to wage cyclicality. This paper contributes to the literature in two key ways. First, it highlights the role of the strategic interactions in aggregate wage adjustment. Specifically, negative external shocks slow down wage adjustment through the firm network. Second, we show that there is a substantial degree of strategic interactions across firms at different levels of the firm wage job ladder.

The rest of the paper is organized as follows. In Section 1, we motivate our empirical approach with a simple model. Section 2 presents the data, discusses the main empirical findings, estimates network formation, and re-estimates strategic interactions with endogenous networks. In Section ??, we develop a general equilibrium model featuring oligopsonistic labor markets and conduct policy experiments. Finally, we offer concluding remarks.

1 Model

To motivate our empirical approach, we develop a general equilibrium model with two main features. First, wage posting model with firms having monopsony power. The model has two predictions. First, competition for labor leads to strategic interaction in wages among firms. Second, the model predicts sluggish wage adjustment following an exchange rate depreciation. Even if a given firm is not directly affected by the exchange rate depreciation, it could avoid adjusting wages due to the strategic interactions with its firm-competitors that are affected by the exchange rate depreciation. An increase in the share of firms that use foreign intermediate inputs in production in a given labor market leads to slower aggregate wage adjustment due to the strategic spillovers in wage setting.

The economy consists of a continuum of labor markets. Each labor market is populated by \mathcal{N} firms and \mathcal{X} types of workers. Each type of workers is represented by a unit mass of workers. We use capital letters to define levels, lowercase letters to define logs of levels, and curved capital letters to define aggregate variables.

1.1 Workers

We assume that there are \mathcal{X} types of workers. Each type $x \in \{1, \dots, \mathcal{X}\}$ is represented by a unit mass of workers of its type. We assume that the firms are grouped into $G + 1$ exhaustive and mutually exclusive sets, $g = 0, 1, \dots, G$. We will denote the set of firms in group g as \mathcal{J}_g . The “unemployment” option, $j = 0$, is the only member of group 0. Each worker i of type x values firm $j \in \mathcal{J}_g$ at period t according to:³

$$\mathcal{V}_{t,ij}^w = \phi_x \log W_{xjt} + \log V_{xj} + \log Z_{jt}\beta_z + \epsilon_{ij},$$

where W_{xjt} stands for the wage offered by firm $j \in \{1, \dots, J\}$ to type x 's worker at period t , and parameter ϕ_x captures the degree of substitution for a worker of type x across firms. The worker of type x also derives unobservable utility V_{xj} from firm j . Firm-specific time-variant characteristics, Z_{jt} , capture how shocks to firms affect workers preferences. In our empirical analysis, we construct shocks to firms based on their exposure to exports and imports. The unemployment option is modeled

³Without loss of generality, we assume the absence of an outside option in the form of being unemployed. The prediction of the model is not sensitive to the assumption given that the main ingredient here is an interaction of firms. Also note that we assume static preferences without loss of generality. The dynamic preferences could be modeled following Rust (1987) and would generate a labor supply function with similar features to the static one.

according to:

$$\mathcal{V}_{t,i0}^w = \log V_{x0t} + \epsilon_{i0},$$

where $\log V_{x0}$ is unobservable utility derived from “unemployment” state by worker of type x . Variable ϵ_{ij} is a generalized extreme value random variable that captures idiosyncratic taste shock of worker i to option $j \in \{0, 1, \dots, J\}$ with correlation of utilities among the firms in the same group is ρ_x , where $0 \leq \rho_x < 1$.

Worker i ' decision rule is described by the following problem:

$$j^* = \operatorname{argmax}_{j \in \{0,1,\dots,J\}} \{\mathcal{V}_{t,ij}^w\} = \operatorname{argmax}_{j \in \{0,1,\dots,J\}} \{\mathcal{V}_{xjt} + \epsilon_{ij}\}, \quad (1)$$

where $\mathcal{V}_{xjt} = \phi_x \log W_{xjt} + \log V_{xj} + \log Z_{jt} \beta_z$ and $\mathcal{V}_{x0t} = \log V_{x0t}$. The expected utility function of the worker has the closed form solution for the distribution of ϵ_{ij} we assume. Following McFadden (1981), it is called the social surplus function and takes the form:

$$\mathcal{V}_{xt}^w = \mathbb{E} \{\mathcal{V}_{t,ij}^w\} = \log \sum_{g=0,1,\dots,G} \left(\sum_{j \in \mathcal{J}_g} \exp \left\{ \frac{\mathcal{V}_{xjt}}{1 - \rho_x} \right\} \right)^{1 - \rho_x}. \quad (2)$$

The above preferences along the assumption of unit mass workers generate labor supply function for firm j by x -type of workers:

$$L_{xjt}(W_{xjt}; \mathbf{W}_{-xjt}) \equiv \frac{\partial \mathcal{V}_{xt}^w}{\partial \mathcal{V}_{xjt}} = \frac{\left(W_{xjt}^{\phi_x} V_{xj} Z_{jt}^{\beta_z} \right)^{\frac{1}{1 - \rho_x}}}{D_g} \frac{D_g^{1 - \rho_x}}{\sum_{g' \in \{0,1,\dots,G\}} D_{g'}^{1 - \rho_x}}, \quad (3)$$

where $D_g = \sum_{k \in \mathcal{J}_g} \left(W_{xkt}^{\phi_x} V_{xk} Z_{kt}^{\beta_z} \right)^{\frac{1}{1 - \rho_x}}$ and \mathbf{W}_{-xjt} as a vector of all wages offered by all firms excluding firm j . Following the McFadden's notations, $\log D_g$ defines the inclusive value of group g . Note that parameter ϕ_x captures the degree of substitution for a given worker across firms. If parameter ϕ_x is zero and all firms offer identical non-wage values then labor supply for each firm is inelastic and equal $1/N$. On the other hand, with parameter ϕ_x being positive, the labor supply function is upward-sloping and features the strategic interactions in wage setting.

1.2 Firms

To model differentiated demand for goods produced by firms, we follow the standard approach in the literature. We separate the firm's block into final good producers and intermediate good producers namely *firms*. Moreover, we separate demand from domestic and foreign customers to define export exposure of firms.

1.2.1 Final good producers

We assume final good producers aggregate goods produced by intermediate good producers and sell them in domestic markets to households and in foreign markets to the rest of the world (RoW). We define variables associated to the RoW with asterisks. Additionally, we assume that production functions for the home and RoW markets might differ in a way that firms' goods take different weights in production of final goods. Specifically, the production function for the home and RoW markets are, respectively:⁴

$$\mathcal{Y}_t = \left(\sum_{j=1}^J \mu_j^{\frac{1}{\eta}} Y_{jt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad (4)$$

$$\mathcal{Y}_t^* = \left(\sum_{j=1}^J \mu_j^{*\frac{1}{\eta}} Y_{jt}^{*\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad (5)$$

where parameters μ_j and μ_j^* represent weights intermediate goods of firm j used in production of final goods for the home and RoW markets, respectively; and parameter η stands for the elasticity of substitution among the intermediate goods.

Following the vast literature on the dominant currency paradigm (see, for example, Gopinath et al. (2020), Mukhin (2022)), we assume that firms sell good in the home and the RoW markets in pesos and dollars, respectively. Normalizing the price of the final goods to one⁵, the demand for goods produced by firms is:

$$Y_{jt} = \mu_j P_{jt}^{-\eta} \mathcal{Y}_t, \quad (6)$$

$$Y_{jt}^* = \mu_j^* P_{jt}^{*-\eta} \mathcal{Y}_t^*, \quad (7)$$

⁴Note that we rule out cyclical markups in product markets by design. The CES production functions feature constant markups.

⁵One peso for domestic goods, and one dollar for RoW final goods.

where P_{jt} and P_{jt}^* are prices of goods produced by firm j invoiced in pesos and dollars, respectively.

1.2.2 Intermediate good producers

We assume that firm j produces goods using labor and foreign intermediate inputs according to production function $F(L_{jt}, M_{jt}) = \tau_j L_{jt}^{1-\alpha_j} M_{jt}^{\alpha_j}$, where variable L_{jt} captures the firm's labor, variable M_{jt} stands for quantity of foreign intermediate inputs used in production, and parameter τ_j is a scalar such that $\tau_j = \alpha_j^{-\alpha_j}$. Parameter α_j captures share of foreign intermediate inputs used in production. Firm's labor, L_{jt} , is an aggregate of labor across all skills such that $L_{jt} = A_j \prod_{x=1}^{\mathcal{X}} L_{xjt}^{\beta_{xj}}$. We assume that foreign intermediate inputs are traded in foreign markets with perfectly elastic supply at one dollar per one unit of input, with the former assumption is motivated by the fact that majority of foreign intermediate inputs are invoiced in dominant currencies. Let us define \mathcal{E}_t as the exchange rate (price of dollar in pesos). Therefore, the problem of the firm can be expressed as:

$$\begin{aligned} \mathcal{V}_{jt}^f = \max & \left\{ P_{jt} Y_{jt} + \mathcal{E}_t P_{jt}^* Y_{jt}^* - \sum_{x=1}^{\mathcal{X}} W_{xjt} L_{xjt} - \mathcal{E}_t M_{jt} \right\} \\ & \text{subject to (3), (6), (7) and } Y_{jt} + Y_{jt}^* \leq F(L_{jt}, M_{jt}) \\ & \{H_{xjk}\}, k \neq j \text{ and } j, k \in \mathcal{J}_g, \end{aligned} \quad (8)$$

where the first two components define peso revenues from sales in domestic and/or foreign markets; the third and fourth terms stand for cost of labor and foreign intermediate inputs in production. The firm maximizes its value function by choosing prices of goods sold to the final good producers, wages to its workers and foreign intermediate inputs subject to the system of labor supply functions (3), demand for goods (6) - (7), and production capacity. Finally, the system $\{H_{xjk}\}_{k \neq j \text{ and } j, k \in \mathcal{J}_g}$ defines strategic interactions in wage setting. We assume that firms strategically interact only with other firms in the same group g , and the system allows us to control for the market structure. Specifically, we assume that firms might compete in quantities of labor (Cournot-Nash assumption), wages (Bertrand-Nash assumption), and collude. The specific system for the above market structures is provided in Appendix D.1.

The log-linear approximation of equilibrium wage at firm j for x -type workers takes the form:

$$\log W_{xjt} = \log \tilde{Y}_{xjt} + (\tilde{\gamma}_j - \tilde{\alpha}_j) \log \mathcal{E}_t + \log \mathcal{M}_{xjt} \left(W_{xjt}, \{W_{xkt}\}_{k \neq j} \right), \quad (9)$$

where variable $\tilde{Y}_{xjt} = \beta_{xj} \frac{\partial L_{jt}}{\partial L_{xjt}}$ defines marginal contribution of x -type labor to overall firm's labor; parameter $\tilde{\gamma}_j$ defines equilibrium exposure of firm j to export and is proportional to $\frac{Y_{jt}^*}{Y_{jt} + Y_{jt}^*}$; parameter $\tilde{\alpha}_j$ defines exposure to foreign intermediate inputs in production and proportional to share of foreign intermediate inputs used in production; variable \mathcal{M}_{xjt} is markdown at firm j for x -type labor such that $\mathcal{M}_{xjt} = \frac{\sigma_{xjt}}{\sigma_{xjt} + 1}$, and variable σ_{xjt} is perceived labor supply elasticity. A closed form solution of the perceived labor supply elasticity depends on a market structure. We derive the closed form solution for market structures with the following assumptions: (i) Bertrand-Nash, (ii) Cournot-Nash, and (iii) Collusion in Appendix D.1. The perceived labor supply elasticity for each case is reported below:

$$\sigma_{xjt} = \begin{cases} \frac{\phi_x}{1-\rho_x} (1 - \rho_x L_{j|xg} - (1 - \rho_x) L_{xj}), & \text{under Bertrand-Nash,} \\ \frac{\phi_x}{1-\rho_x} \left(1 - \frac{L_{j|xg}(\rho_x + (1-\rho_x)L_{xg})}{1 - (1-L_{j|xg})(\rho_x + (1-\rho_x)L_{xg})} \right), & \text{under Cournot-Nash} \\ \phi_x (1 - L_{xg}). & \text{under Collusion} \end{cases} \quad (10)$$

Equation (9) demonstrates that markdown is a function of a firm's own wage and wages of its competitors. A linear approximation of the above equation yields us wage adjustment in firm j as a change in the logarithm of exchange rate Δe_t , and change in the logarithm of wages of firm-competitors, Δw_{xjt} with $k \neq j$:

$$\Delta \log W_{xjt} \approx \sum_{k \neq j} \mathcal{M}_k \Delta \log W_{xkt} + (\tilde{\gamma}_j - \tilde{\alpha}_j) \Delta \log \mathcal{E}_t + \sum_{k \neq j} \frac{\partial \mathcal{M}}{\partial \tilde{\gamma}_k} (\tilde{\gamma}_k - \tilde{\gamma}_j) \Delta \log \mathcal{E}_t + \sum_{k \neq j} \frac{\partial \mathcal{M}}{\partial \tilde{\alpha}_k} (\tilde{\alpha}_k - \tilde{\alpha}_j) \Delta \log \mathcal{E}_t, \quad (11)$$

with variable \mathcal{M}_k defining $\partial \mathcal{M} / \partial w_{xkt}$. With the labor supply function (3), \mathcal{M}_k has an explicit expression and proportional to the equilibrium share of employment by firm k , N_{xkt} .

It is important to note that the partial equilibrium model generates two main predictions. First, the model demonstrates that an exchange rate depreciation leads to a stagnant wage adjustment by directly affected firms. Directly affected firms pass through the cost shock to their own wages. Second, even if a given firm is not exposed to the exchange rate fluctuation via the usage of foreign intermediate inputs, an exchange rate depreciation could feedback through firm-competitors that are directly affected. The strategic interaction in wage setting could lead to spillovers among firms via a network of firm-competitors. This channel is novel to the literature.

1.3 Closing the Model

1.3.1 Households

We assume that the household sector is represented by a family of workers. At the beginning of each period, workers choose firms according to their preferences; however, at the end of each period, they bring all wages earned back to the family and decide how to consume and accumulate wealth together. We assume that the household sector has the following utility function:

$$\mathcal{V}_t(\mathcal{B}_{t-1}^*) = \max \{ \log \mathcal{C}_t + \mathbb{E}_t \mathcal{V}_{t+1}(\mathcal{B}_t^*) \}, \quad (12)$$

where \mathcal{C}_t is a composite consumption index of domestic and foreign goods, \mathcal{C}_{Ht} and \mathcal{C}_{Ft} , respectively, such that $\mathcal{C}_t = \left(\frac{\mathcal{C}_{Ht}}{\omega}\right)^\omega \left(\frac{\mathcal{C}_{Ft}}{1-\omega}\right)^{1-\omega}$ with parameter ω defining share of households' spending on domestic goods; variable \mathcal{B}_{t-1}^* defines net foreign assets carried over from period $t - 1$. The households maximize the utility function subject to the budget constraint:

$$\mathcal{P}_t \mathcal{C}_t + \mathcal{E}_t \mathcal{B}_t^* = \sum_{j=1}^{\mathcal{J}} \sum_{x=1}^{\mathcal{X}} W_{xjt} L_{xjt} + \mathcal{E}_t \mathcal{O}_t + (1 + i_{t-1}^*) \mathcal{E}_t \mathcal{B}_{t-1}^*, \quad (13)$$

where $\mathcal{P}_t \mathcal{C}_t \equiv \mathcal{C}_{Ht} + \mathcal{E}_t \mathcal{C}_{Ft}$ represents households' expenditure on domestic and foreign goods; i_t^* stands for interest rate on net foreign assets, and \mathcal{O}_t is endowment of oil in dollars.

1.3.2 Open Economy component

Before closing the model, we need to define how oil endowment, \mathcal{O}_t , and interest rate on net foreign assets, i_t^* , are determined. First, following the open economy literature, we assume that the oil endowment is determined according to:

$$\log \mathcal{O}_t - \log \bar{\mathcal{O}} = \rho_o (\log \mathcal{O}_{t-1} - \log \bar{\mathcal{O}}) + \epsilon_{ot}, \quad (14)$$

where $\bar{\mathcal{O}}$ represents the steady state level of oil endowment; parameter ρ_o captures persistence of the oil endowment shocks, ϵ_{ot} . Second, following Schmitt-Grohe & Uribe (2003), we assume that the interest rate on net foreign assets is determined by:

$$i_t^* = \bar{i}^* + \psi \frac{\mathcal{B}_t^* - \bar{\mathcal{B}}^*}{\bar{\mathcal{B}}^*} + \epsilon_{i^*t}, \quad (15)$$

where parameter ψ represents how the interest on net foreign assets changes with respect to deviations of the net foreign asset position from its steady state value. The net foreign interest rate is also subject to shocks, ϵ_{i^*t} , in the line with Gabaix & Maggiori (2015) and Itskhoki & Mukhin (2021).

1.4 Equilibrium

This subsection defines a competitive equilibrium.

DEFINITION 1 *A competitive equilibrium is a set of final good producers' prices $\mathbb{P}_t = \{P_{jt}\}_{j=1}^{\mathcal{J}}$ and $\mathbb{P}^* = \{P_{jt}^*\}_{j=1}^{\mathcal{J}}$, firms' wages $\mathbb{W}_{xt} = \{W_{xjt}\}_{j=1}^{\mathcal{J}}$ for each type x , decision rules $\{L_{xjt}(\mathbb{W}_{xt})\}_{x,j}$, $\mathbb{Y}_t = \{Y_{jt}\}_{j=1}^{\mathcal{J}}$, $\mathbb{Y}_t^* = \{Y_{jt}^*\}_{j=1}^{\mathcal{J}}$, household's choices $\{\mathcal{C}_t, \mathcal{C}_{Ht}, \mathcal{C}_{Ft}, \mathcal{B}_t^*\}$ such that, given firms' productivities $\mathbb{A} = \{A_j\}_{j=1}^{\mathcal{J}}$, and international shocks \mathcal{O}_t and i_t^* ,*

1. *The firms' wages $\mathbb{W}_{xt} = \{W_{xjt}\}_{j=1}^{\mathcal{J}}$ solve the intermediate good producer's problem (9),*
2. *Workers choose firms according to (1) that results in the labor supply (3).*
3. *The demand for each intermediate good producer's output from the final good producers Y_{jt} and Y_{jt}^* is equal to the supply of that firm's output $F_j(L_{jt}, M_{jt})$,*
4. *Household smooths consumption; that is: $1 = (1 + i_t^*) \beta \mathbb{E}_t \left\{ \frac{\mathcal{C}_t}{\mathcal{C}_{t+1}} \frac{\mathcal{P}_t}{\mathcal{P}_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\}$,*
4. *The home goods market clears; that is: $\mathcal{Y}_t = \mathcal{C}_{Ht}$,*
5. *The balance of payment holds; that is: $\mathcal{B}_t^* = \mathcal{Y}_t^* + \mathcal{O}_t - \mathcal{M}_t - \mathcal{C}_{Ft} + (1 + i_{t-1}^*) \mathcal{B}_{t-1}^*$, where $\mathcal{M}_t = \sum_{j=1}^{\mathcal{J}} M_{jt}$ and $\mathcal{C}_{Ft} = (1 - \omega) \mathcal{E}_t^{-\omega} \mathcal{C}_t$,*
6. *The labor markers clear*

$$\sum_{j=1}^{\mathcal{J}} L_{xjt} = 1 \text{ for each } x.$$

Condition (i) requires that the firms' employment decision rules to be optimal given wages, productivities and international shocks. Condition (ii) requires the workers' decision be optimal given wages and external shocks to firms. Condition (iii) requires the demand for firms' output is equal to supply. Finally, conditions (iv)-(vi) require the markets to clear. The home goods are consumed by households. The . The labor market is defined by workers who supply labor and firms that demand it.

1.5 Discussion

Before moving to the empirical section, we provide mapping between the partial equilibrium model and reduced-form equation we aim to estimate. The linear approximation of equilibrium wages motivate the following reduced-form regression to test for the strategic interactions in wage setting:

$$\begin{aligned} \Delta \log W_{xjt} = & \lambda_x \sum_{k \neq j} \pi_{j,k} \Delta \log W_{xkt} + \beta_1^{\text{exp}} \tilde{\gamma}_j \Delta \log \mathcal{E}_t + \beta_1^{\text{imp}} \tilde{\alpha}_j \Delta \log \mathcal{E}_t + \\ & \beta_2^{\text{exp}} \sum_{k \neq j} \pi_{j,k} (\tilde{\gamma}_k - \tilde{\gamma}_j) \Delta \log \mathcal{E}_t + \beta_2^{\text{imp}} \sum_{k \neq j} \pi_{j,k} (\tilde{\alpha}_k - \tilde{\alpha}_j) \Delta \log \mathcal{E}_t + u_{xjt}, \end{aligned} \quad (16)$$

where variable $\Delta \log W_{xjt}$ is a change in logarithm of wage of firm j from period $t - 1$ to t , and variable $\Delta \log \mathcal{E}_t$ represents a change in logarithm of exchange rate between periods $t - 1$ and t . Matrix $\pi_{j,k}$ represents weights of firm-competitors to firm j . That matrix has a zero diagonal and is row-normalized so that the sum of all elements in a given row is equal to one. According to the model, matrix π might be constructed from shares of employment by firms within a given labor market; however, in reality, it is hard to identify firm-competitors for two main reasons. First, the definition of labor market borders is vague. Even when firms are within the same industry or close in terms of geographic location, they do not necessarily compete with each other for labor. For example, one can think of a firm hiring only managers, and another firm hiring only technicians. Second, labor markets might be defined in terms of observable or unobservable worker characteristics. For instance, one firm could hire only female workers, while another – only male ones. We alleviate this problem using actual flows of workers between firms to identify direct firm-competitors.

The first component of equation (16) defines the contribution of the strategic complementarity in wage setting, which is also called endogenous peer effects in the network and peer effect literature. The second component represents the contribution of exogenous shocks to wages that in our framework comes from the exposure to exchange rate fluctuations. The last component captures the exogenous peer effects in line with the peer effects literature. Note that the last term represents the contribution of shock to firm-competitors in relative terms. That is, firm j is affected by shocks to firm-competitors if it is differentially exposed to them relative to their competitors.

[add estimation of labor supply elasticity -]

2 Empirical investigation

The empirical section is organized as follows. First, we describe the main datasets, providing an overview of their coverage, summary statistics, and the construction of key variables for the analysis. Second, we estimate firm-to-firm workers' flows and analyze how our estimation procedure could treat the endogenous nature of networks, demonstrating how our estimation procedure accounts for it. We also examine the implications for aggregate wage adjustment, discussing how the network structure propagates external shocks throughout the firm-to-firm network. Third, we detail our approach to identifying strategic interactions in wage setting and present the main results. Finally, we outline the limitations we encounter in our empirical analysis.

2.1 Data

In our estimation, we rely extensively on two main data sources. First, we utilize a dataset on social security payment reports from the *Planilla Integrada de Liquidación de Aportes (PILA)* system. The *PILA* system is similar to the Social Security Administration dataset in the United States, as it records both employee and employer tax identifiers. However, unlike the U.S. system, the *PILA* system provides monthly records along with the number of days worked at a given firm. This additional level of detail proves useful in two ways. First, it allows us to analyze intensive pay measures, such as daily wage, which helps eliminate potential measurement error. Second, for workers with multiple jobs within a given period, we can assign a main job based on the number of days worked at each firm. One limitation of the *PILA* dataset is that it does not distinguish different establishments (if any) within the same firm. As a result, we define firms at the level of the reported employer tax identifier, acknowledging this constraint in the data.⁶

Second, we use the input-output table at the regional and two-digit industry level, including the foreign sector, as constructed by the *Departamento Administrativo Nacional de Estadística (DANE)*. This table defines firms' exposure to exports and foreign intermediate inputs in production across regions and industries. Figures B.2 and B.3 present heat maps showing the exposure measures of metropolitan areas to export and foreign intermediate inputs in production, where the exposure measures for each area is weighted by firms' employment in two-digit industries. The heat maps reveal

⁶It is a commonly used approach to identify "firms" using employer tax identifiers in social security datasets, as this is often the most reliable identifier available. For example, see Song et al. (2018) for a detailed application of this approach.

significant variation in both exposure to exports and foreign intermediate inputs in production across metropolitan areas, with notable differences even within the same area, highlighting the heterogeneity in external exposure to exports and foreign inputs in production.

The *PILA* system uses official identifiers for both employers and employees. The employer's identifier, known as *NIT*, *Numero Unico de Identificacion Tributaria*, is equivalent to the employer identification number in the United States. The employee's unique identifier, called *cedula*, is comparable to the U.S. Social Security number. We restrict our sample to the years 2009-2018 and retain only one observation per worker per year, selecting the primary employer based on the number of days worked. We keep only employees earning above minimum wage. We also retain observations in one of the 62 metropolitan areas reported in Table A.2, with reported industry and age.

Over the ten-year period, the *PILA* system provides over 61 million worker-year observations, over 13 million unique employees, around 0.7 million unique firms and over 0.9 million MSA-firms. The sample size and summary statistics are presented in Table A.1. The data spans over 62 metropolitan areas and 61 two-digit industries. Table A.2 details the spatial coverage for 42 metropolitan areas, accounting for over 98% of *PILA* worker-year observations from 2009 to 2018. In our empirical analysis, we concentrate on top-10 metropolitan areas that constitute around 80% of formal employment in Colombia. Industry coverage is provided in Tables A.3 and A.4.

Network matrix. Matrix π is a zero-diagonal \mathcal{J} -by- \mathcal{J} matrix, representing the weights associated with firm-competitors for a given firm. It is constructed based on worker flows between firms as follows:

$$\pi_{j,k} = \frac{\text{Flow}_{j \leftrightarrow k}}{\sum_{i=1: i \neq j}^{\mathcal{J}} \text{Flow}_{j \leftrightarrow i}}, \quad k \neq j,$$

where $\text{Flows}_{j \leftrightarrow k}$ represents the total number of workers who moved from firm j to firm k and vice versa with a short period of time between the two jobs. We consider only moves that happen within a 30-days interval assuming that they represent job-to-job flows. This interval is in the line with the U.S. Bureau of Labor Statistics that runs the Current Population Survey on a monthly basis. Similarly, Haltiwanger, Hyatt, Kahn & McEntarfer (2018) target only job flows with a period of time between two jobs being less than one month. Figure B.4 demonstrates transitions as a share of employment with a period of time between two jobs being less than two weeks, three weeks, one month, and transitions without a restriction on time being in a transition.

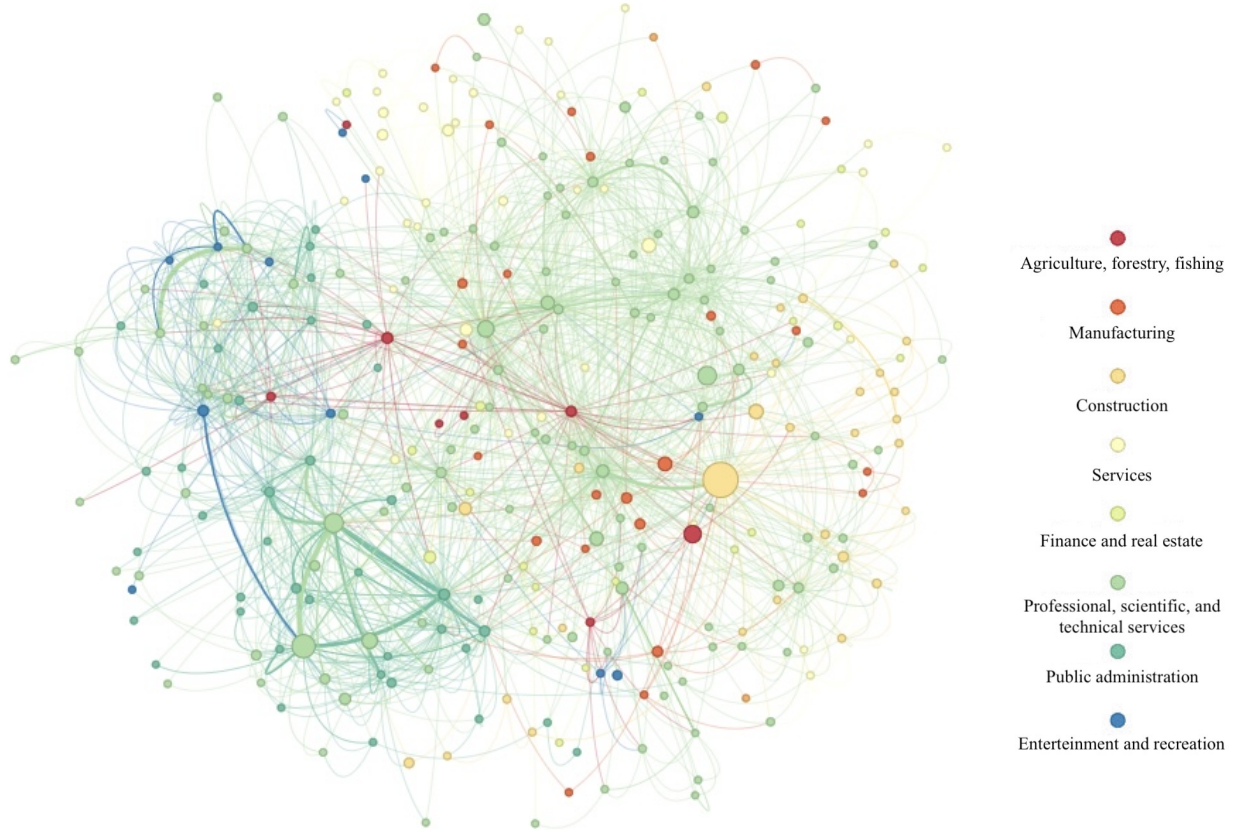
The element (j, k) of matrix π represents the share of workers who left (join) firm j for (from) competitor k relative to all other firms with whom firm j has positive flows. Several points are worth noting. First, the flow of workers between any two firms is directed, meaning that the number of workers moving from one firm to another is not necessarily equal to the number moving in the opposite direction. Second, the construction of the firm-competitor matrix aligns with how workers value firms in the partial equilibrium model, identifying potential firm-competitors through their movements. Third, the sum of each row in matrix π equals one. Therefore, the product of the weights in matrix π and the wage changes of firm-competitors can be interpreted as the average wage change of a firm's competitors. This captures how wage adjustments among a firm's competitors, weighted by the proportion of workers who moved between them, influence the firm's own wage-setting decisions. In Figure 1, we illustrate job flows between firms. The figure demonstrates two important features of job flows in our sample. First, job flows are sparse in its nature. Workers do not flow from one to all firms but rather to a small subset of firms in a given metropolitan area. Second, we observe that workers flow between firms not necessarily in the same industry. This indicates that less affected industries still could be affected by shocks via firm-competitors in more affected industries.

Firm's wage adjustment. We construct the firm's wage adjustment variable using worker-level wage data, using wage adjustment of job-stayers. By definition, job-stayers remain with the same primary employer across the periods of interest.

For each firm, we consider all job-stayers and calculate the wage change for each worker i , who is employed by firm j in 2014 and period t (where $t = 2015, \dots, 2018$), as $\Delta w_{ij} = w_{ijt} - w_{ij2014}$. Calculating the wage change at the worker level helps eliminate time-invariant unobservable components of wage compensation specific to each worker. Once we have individual wage changes for job-stayers, we define the firm's wage adjustment by averaging the wage changes across all job-stayers in that firm. We assess wage adjustments over different time intervals to capture strategic interactions in wage setting across various horizons. We refer to the one-year horizon as the short run and the four-year horizon as the medium run.

External shocks. For each firm j , we define the firm's exposure to external shock, denoted as $\Delta z_j =$

Figure 1. Job flows: Illustration



Notes: The source of data is authors' computation and the *PILA*. The figure is based on Cartagena metropolitan area.

$[z_j^1, z_j^2]$, where:

$$\Delta z_j^1 = (\log \mathcal{E}_t - \log \mathcal{E}_{2014}) \times \text{Export Exposure}_{RI(j)}$$

$$\Delta z_j^2 = (\log \mathcal{E}_t - \log \mathcal{E}_{2014}) \times \text{Import Exposure}_{RI(j)}$$

where \mathcal{E}_t is the average exchange rate in year t ($t = 2015, \dots, 2018$), and $\text{Exposure}_{RI(j)}$ represents the share of foreign intermediate inputs in production used in the region and 2-digit industry associated with firm j .

2.2 Identification

Motivated by the partial equilibrium model, we identify strategic interactions in wage setting using the linear-in-mean specification from the peer effect literature *à la* Manski (1993):

$$\Delta w_j = \zeta_m + \lambda \sum_{k=1}^{\mathcal{N}} \mathcal{W}_{j,k} \Delta w_k + \Delta z_j \beta_1 + \sum_{k=1}^{\mathcal{N}} \mathcal{W}_{j,k} (\Delta z_k - \Delta z_j) \beta_2 + u_j, \quad (17)$$

where ζ_m represents either a constant or metropolitan area fixed effects, and Δw_j denotes the average wage adjustment for job-stayers at firm j . The variable Δz_j captures the exposure firm j 's to external shocks at the region and 2-digit industry level in which firm j operates. We assume that u_j is distributed identically and independently across firms. We estimate strategic interactions in wage setting over four different time intervals (2014 – 15, ..., 2014 – 18), allowing us to infer the contribution of these interactions over both short and medium-run horizons. Since matrix \mathcal{W} is a row-normalized, the parameter λ can be interpreted as the percentage change in a firm's wage adjustment resulting from a one-percent increase in the average wage of its competitors.

We identify all parameters in (17), using the generalized method of moments (GMM) procedure for estimating peer effects, as developed by Kuersteiner & Prucha (2020). The procedure leverages both linear and quadratic moments to identify endogenous and exogenous peer effects. The motivation for the linear moments comes from the following observation.⁷ If matrix $I - \lambda \mathcal{W}$ is invertible, then the conditional weighted average of wage changes of firm-competitors is a non-linear function of $\mathcal{W}^{s+1} \Delta z$, $s = 0, \dots$, and the composite parameter β^* , which itself is a function of λ , β_1 , and β_2 :

$$\mathbb{E}[\mathcal{W} \Delta w | \Delta z] = \mathcal{W} (I - \lambda \mathcal{W})^{-1} \Delta z \beta^* = \sum_{s=0}^{\infty} \lambda^s \mathcal{W}^{s+1} \Delta z \beta^*.$$

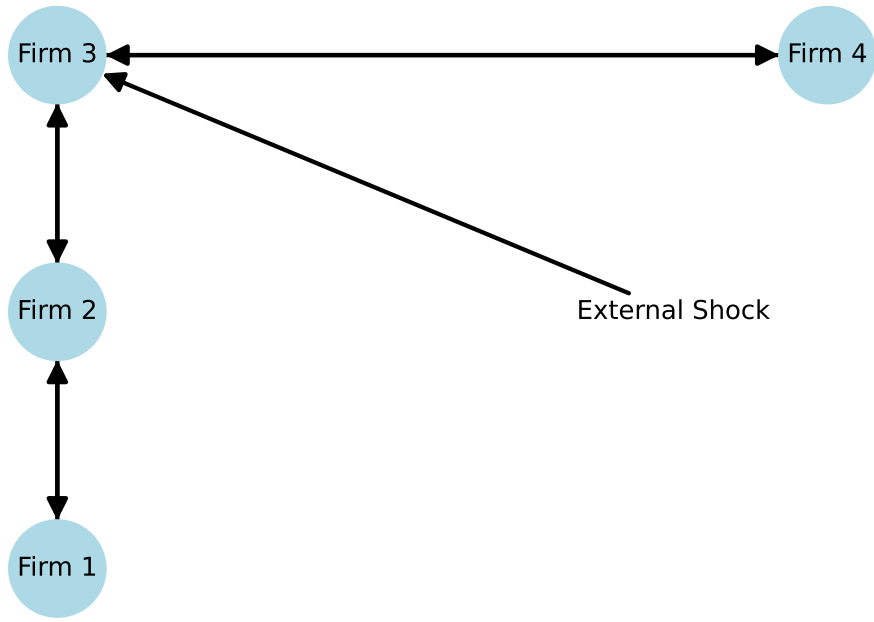
This suggests that a set of linear moments can be constructed using the linear independent columns $\{\mathcal{W}^{s+1} \Delta z, s = 0, \dots\}$ as instruments. On the other hand, the quadratic moment can be constructed noting that the conditional variance-covariance matrix of idiosyncratic disturbances can be written as

$$\mathbb{V}\mathbb{C}[\Delta w | \Delta z] = (I - \lambda \mathcal{W})^{-1} (I - \lambda \mathcal{W})^{-1} \sigma_u^2 = \sum_{s=0}^{\infty} \sum_{\tau=0}^{\infty} \lambda^{s+\tau} \mathcal{W}^s \mathcal{W}'^{\tau} \sigma_u^2.$$

In Figure 2, we illustrate the intuition behind identification using linear and quadratic moments

⁷Equation (17) can be represented in the matrix notation as $\Delta w = \zeta + \lambda \mathcal{W} \Delta w + \Delta z \beta_1 + (\mathcal{W} - I) \Delta z \beta_2 + u$, where Δw and Δz are \mathcal{N} -by-1 column vectors, and \mathcal{W} is an \mathcal{N} -by- \mathcal{N} matrix.

Figure 2. Illustration of Identification: a Simplified Example



under the assumption of independent and identically distributed errors. The example consists of four firms within a labor market represented as a graph, where nodes correspond to firms and arrows indicate worker flows, reflecting the direction of competition among firms. This competition can often be attributed to the physical proximity of firms; for instance, firms 3 and 4 are located at the northern boundary of the market, firm 2 is centrally situated, and firm 1 is in the southern part of the market. Our question is how a firm adjust wage to its firm-competitors. First, external shocks to firms that are not direct firm-competitors for a given firm can serve as external instrumental variables (IVs). In this example, an external shock to firm 3 can be used as an IV for firm 1's wage adjustment, which forms the basis for linear moments in our identification strategy. Second, wage adjustment by higher order competitors (indirect competitors) for a given firm can also serve as IVs. For example, wage adjustments by firm 4 influence firm 1's wage adjustment through the network structure, providing the motivation for quadratic moments.

In our setting, the final set of linear and quadratic moment conditions are:

$$\begin{aligned}\mathbb{E} \left[h^{p'} \tilde{u} \right] &= 0, \\ \mathbb{E} \left[\tilde{u}' \mathcal{A}^q \tilde{u} \right] &= 0,\end{aligned}$$

where $h = (h^1, \dots, h^4) = (\zeta, \Delta z, \mathcal{W}\Delta z, \mathcal{W}^2\Delta z)$, $\tilde{u} = \Omega u$, $\mathcal{A}^1 = \frac{\mathcal{W} + \mathcal{W}'}{2}$, and $\mathcal{A}^2 = \mathcal{W}'\mathcal{W} - \text{diag}(\mathcal{W}'\mathcal{W})$, $\Omega = \text{diag}(\omega)$, and ω is a vector of weights. We proxy weights with number of job-stayers in a given firm.⁸ The linear moments are based on internal and external instruments. The external instrument $\mathbb{E}[h^{4'}\epsilon] = 0$ is motivated by the peer effects estimation literature with the idea that external shocks to competitors of competitors could serve as instruments for the average wage of firm competitors.⁹ The quadratic moments state orthogonality of idiosyncratic shocks across space.

2.3 Network estimation

In reality, the network formation formed through job-to-job flows is endogenous in nature. Therefore, we need to correct for its endogeneity. This section demonstrates how we estimate network formation among firms – that is, how firms interact with each other. We estimate the network formation for two reasons. First, network formation is an endogenous process; therefore, a researcher should be careful when identifying peer effects. Kuersteiner & Prucha (2020) propose a robust identification with potentially endogenous network formation. The authors demonstrate that the construction of moment conditions with estimated network formation model can deliver unbiased estimates of peer effect coefficients with potentially endogenous networks. Second, the estimation of networks allows us to understand their homophily. A growing literature on estimation of networks includes de Paula (2017), Graham (2016, 2020), and others. At the same time, a bulk of literature on networks shows that networks could be a source of shocks' propagation.

Following the seminal framework by Silva & Tenreyro (2006), we estimate network formation using the pseudo-maximum likelihood approach. Silva & Tenreyro (2006) demonstrate appealing properties of the pseudo-maximum likelihood with data having many zeros. This is the salient feature of firm-to-firm worker flows with most majority of firm-pairs having zero flow of workers between each other. We assume that flow of workers between firms is drawn from the Poisson distribution with the conditional mean:

$$\mathbb{E}(M_{j \rightarrow k, t} | \chi_{jk, t}) = \exp(\chi_{jk, t}), \quad (18)$$

⁸The weighting with number of employees in the initial period does not change the results.

⁹Bramoullé et al. (2009) define conditions under which peer effects could be identified using shocks to peers of the peers as an instrument.

with

$$\chi_{jk,t} = \iota + \gamma_1^{\text{empl}} \log(\text{empl}_j) + \gamma_2^{\text{empl}} \log(\text{empl}_k) + \sum_s |x_j^s - x_k^s| \times \psi_x^s + \sum_s (z_{kt}^s - z_{jt}^s) \times \psi_z^s,$$

where ι is a set of fixed effects; variables $\log(\text{empl}_j)$ and $\log(\text{empl}_k)$ capture the size of firm “sender” (firm j) and firm “receiver” (firm k). To determine homophily of the network formation, we also include observable and unobservable characteristics of workers. The set of observable workers’ characteristics include age and sex - we construct variables of average age of workers and share of male workers in a given firm. The set of unobservable characteristics is worker pay components following Abowd et al. (1999).¹⁰ To have a proxy for worker pay component at the firm level, we take an average of worker pay components for each firm. We chose an absolute value of difference between the observable characteristics of workers and the unobservable firm and worker characteristics to measure “proximity” of firms.

The estimation results of the pseudo-maximum likelihood is reported in Table 1.¹¹ We run three specifications to test how flow of workers respond to external shocks in more/less exposed firms. Across all three specifications, two empirical findings stand out. First, the network formation demonstrates a substantial degree of homophily in terms of observable and unobservable worker characteristic. That evidence highlights that firms employ workers of similar age, gender, and skills. Second, we find that more exposed firms to imports (exports) experience a decline in inflow of workers (an increase in inflow of workers - although not statistically significant). Both results provide important insights for identification of strategic interactions with the endogenous network formation.

2.4 Identification with endogenous networks

Using the estimates from the flow of workers between 2013 to 2018, we predict networks in consecutive years in the following way:

$$M_{j \rightarrow k,t}^* = \exp(\chi_{jk,t}^*) \times d_{jk}, \quad (19)$$

¹⁰The details of AKM estimation is provided in appendix C.1 and estimates are reported in appendix A.

¹¹We also estimate networks for the six largest metropolitan areas (Bogota, Medellin, Cali, Barranquilla, Cartagena, Bucaramanga) separately, and the results are reported in Tables A.6 and A.7.

Table 1. Network estimation

	(1)	(2)	(3)
$\log(\text{empl}_j)$	0,162*** (0,002)		
$\log(\text{empl}_k)$	0,158*** (0,010)	0,197*** (0,028)	
worker pay component	-0,365*** (0,014)	-0,798*** (0,016)	-1,206*** (0,017)
age	-0,025*** (0,001)	-0,050*** (0,001)	-0,075*** (0,001)
share of males	-1,337*** (0,014)	-1,623*** (0,017)	-1,906*** (0,020)
exposure to export	0,017 (0,027)	0,110*** (0,031)	
2014 \times exposure to export	-0,007 (0,041)	-0,007 (0,043)	-0,008 (0,042)
2015 \times exposure to export	-0,025 (0,037)	-0,026 (0,038)	-0,028 (0,038)
2016 \times exposure to export	0,060* (0,036)	0,065* (0,037)	0,065* (0,037)
2017 \times exposure to export	0,062 (0,039)	0,066 (0,041)	0,067 (0,041)
2018 \times exposure to export	0,063* (0,038)	0,067* (0,039)	0,067* (0,039)
exposure to import	0,695*** (0,059)	-0,599*** (0,080)	
2014 \times exposure to import	-0,065 (0,086)	-0,080 (0,103)	-0,078 (0,099)
2015 \times exposure to import	-0,160** (0,076)	-0,200** (0,090)	-0,194** (0,087)
2016 \times exposure to import	-0,219*** (0,075)	-0,262*** (0,090)	-0,256*** (0,086)
2017 \times exposure to import	-0,149** (0,076)	-0,176* (0,091)	-0,172** (0,087)
2018 \times exposure to import	-0,249*** (0,0790)	-0,297*** (0,0941)	-0,290*** (0,0901)
MSA fixed effects	✓		
Firm j fixed effects		✓	
Firm $j \times k$ fixed effects			✓
Year fixed effects	✓	✓	✓
# of firm pairs	20,351,838	20,351,838	20,351,838

Notes: Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

with

$$\chi_{jk,t}^* = \hat{\iota} + \hat{\gamma}_1^{\text{empl}} \log(\text{empl}_j) + \hat{\gamma}_2^{\text{empl}} \log(\text{empl}_k) + \sum_s |x_j^s - x_k^s| \times \hat{\psi}_x^s + \sum_s (z_{kt}^s - z_{jt}^s) \times \hat{\psi}_z^s,$$

where variable d_{jk} is an indicator whether there is a flow of workers between firm j and k between 2013 to 2018. Vector X includes the following list of variables: average worker pay component, average age of workers, and share of females in a given firm. One potential counterpart to direct flows of workers might be indirect flow of workers through firm-competitors that are connected to a given firm. First, the distance in worker characteristics is a weighted distance between firm j and k in terms of average worker pay components and share of males. Firms that are alike have smaller distance.

With predicted flows of workers between firms, we construct the network matrix following the previous logic

$$\mathcal{W}_{j,k}^* = \frac{M_{j \rightarrow k}^*}{\sum_{i=1: i \neq k}^{\mathcal{N}} M_{j \rightarrow i}^*}, k \neq j,$$

where firm j allocates weight $\mathcal{W}_{j,k}^*$ to firm k using predicted flow of workers from firm j to firm k relative to all other firm-competitors of firm j . Using predicted weight matrix \mathcal{W}^* , we construct linear and quadratic moments as

$$\begin{aligned} \mathbb{E} \left[h^{*p'} \tilde{u} \right] &= 0, \\ \mathbb{E} \left[\tilde{u}' \mathcal{A}^{*q} \tilde{u} \right] &= 0, \end{aligned}$$

where $\tilde{u} = \Omega u$, and collection of vectors $h^* = (h^{*,1}, \dots, h^{*,4}) = (\iota, \Delta z, \mathcal{W}^* \Delta z, \mathcal{W}^{*2} \Delta z)$, $\mathcal{A}^{*1} = \frac{\mathcal{W}^* + \mathcal{W}^{*'}}{2}$, and $\mathcal{A}^{*2} = \mathcal{W}^{*'} \mathcal{W}^* - \text{diag}(\mathcal{W}^{*'} \mathcal{W}^*)$, and $\Omega = \text{diag}(\omega)$ with ω is a vector of weights. As before, we proxy weights with number of job-stayers in a given firm.

In the context of identifying peer effects within potentially endogenous networks, the corrected network approach addresses the endogenous nature of network formation, as captured in the network estimation. Conditional on having of worker flows from firm j to firm k , weight matrix \mathcal{W} allocates higher weight toward firm-competitor k with firm-competitors that are larger in size, positioned higher up in the wage job ladder, and employ workers with similar observable and unobservable characteristics to those at firm j . If firms are larger in size and located at the top of the wage job ladder are

affected the most by the external shocks, that could lead to the sluggish aggregate wage adjustment triggered by the largest employers in local labor markets.

The first set of results is presented in Table 2, which reports the estimated parameters of the main regression at the economy-wide level, covering 10 metropolitan areas. The table examines wage adjustments over time horizons ranging from one year (2014-15) to four years (2014-18). We interpret a one-year interval as capturing the short-run adjustment, while a four-year interval reflects the medium-run adjustment. For example, in column 1, we observe changes of wages for job-stayers between 2014 and 2015 for 162,577 firms. Across all columns, the results consistently highlight the presence of strategic interactions in wage setting among firms for job-stayers. All coefficients for the weighted average change in wages of job-stayers of firm-competitors are statistically significant. At the same time, all columns demonstrate that the exogenous peer-effects have statically and economically negative effects on wage adjustment. That is, conditional on firm's exposure to external shocks, firms tend to avoid adjusting wages when their firm-competitors face external shocks.

Table 2. Strategic interactions and wage adjustment: Economy level

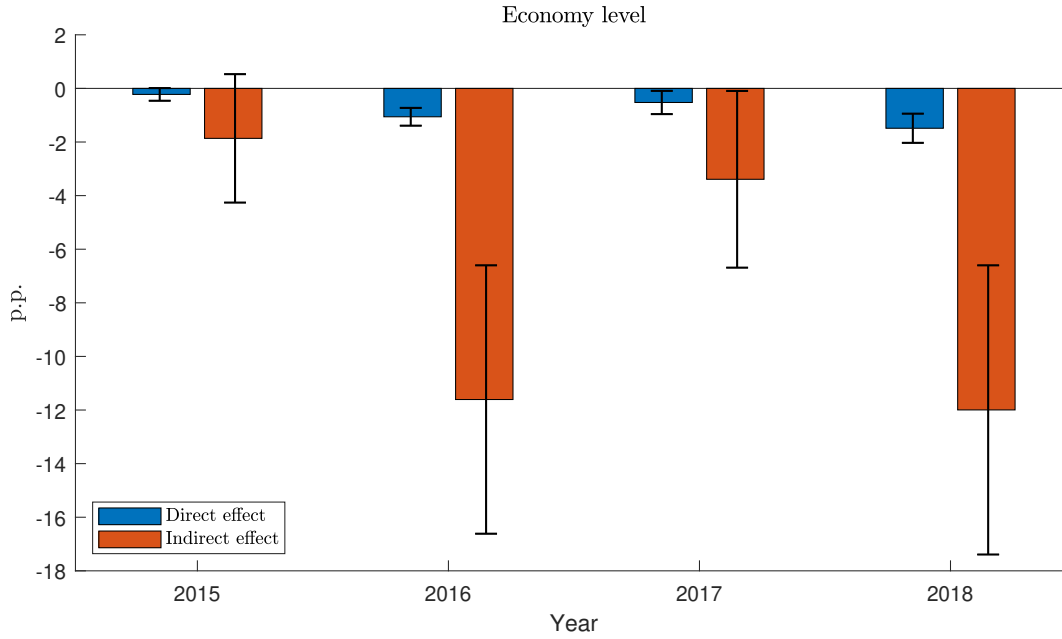
	2014-15	2014-16	2014-17	2014-18
$\mathcal{W}\Delta w$	0.235*** (0.034)	0.217*** (0.022)	0.261*** (0.016)	0.346*** (0.014)
Δz^{exp}	0.007 (0.037)	0.002 (0.064)	0.257*** (0.085)	0.254*** (0.070)
Δz^{imp}	-0.228*** (0.078)	-0.875*** (0.135)	-1.263*** (0.177)	-1.595*** (0.150)
$\mathcal{W}\Delta z^{\text{exp}} - \Delta z^{\text{exp}}$	0.023 (0.038)	0.019 (0.066)	0.236*** (0.088)	0.234*** (0.072)
$\mathcal{W}\Delta z^{\text{imp}} - \Delta z^{\text{imp}}$	-0.263*** (0.082)	-1.007*** (0.142)	-1.454*** (0.188)	-1.927*** (0.160)
MSA FEs	✓	✓	✓	✓
# of firms	162,577	133,817	110,233	98,549
# of balanced work-ers	3,410,686	2,990,038	2,690,508	2,561,246

Notes: Standard errors are in parenthesis. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The table presents estimates for the entire economy with the ten largest metropolitan areas. The table demonstrates wage adjustment estimates from one-year (2014-15) to four-year horizons (2014-18).

2.5 Implications to aggregate wage dynamics

An interesting question is whether strategic interactions in wage setting lead to sizable effects in aggregate wage adjustment. To predict the propagation of external shocks to aggregate wage dynamics,

Figure 3. Contribution of indirect external shocks via network



Notes: The source of data is authors' computation. The bars represent contribution of external shocks to aggregate metropolitan wage adjustment through the direct and indirect channels. The contributions are computed for the entire economy (10 largest metropolitan areas) over 2015 to 2018 years. Standard errors are computed using the delta method.

we decompose (17) in the form and combine predicted wage changes with weights of firms approximated with their employment, ω ,¹²

$$\mathbb{E}[\omega' \Delta w | \iota, \Delta z] = \frac{\hat{l}_m}{1 - \hat{\lambda}} + \underbrace{\omega' \Delta z \hat{\beta}_1}_{\text{Direct Effect}} + \underbrace{\sum_{s=0}^{\infty} \left[\hat{\lambda}^{s+1} \omega' \mathcal{W}^{s+1} \Delta z \hat{\beta}_1 + \hat{\lambda}^s \omega' \mathcal{W}^s (\mathcal{W} - I) \Delta z \hat{\beta}_2 \right]}_{\text{Indirect Effect}},$$

where ω and Δw are $\mathcal{N} \times 1$ vectors. The equation above decomposes the contribution of the external shocks to firms through the direct and indirect channels. As mentioned earlier, the indirect channel is fully operates via the network. To estimate the contribution of both channels, we use the identified parameters using the linear and quadratic moment conditions. The above decomposition gives us the contribution of external shocks directly via firms as well as indirectly via the network of firm-competitors in the interaction in wage setting. Figure 3 summarizes the contribution of indirect external shocks over 2015 and 2018 years. The figure demonstrates that strategic interactions in wage setting with a combination of negative external shocks could lead to sufficient sluggish aggregate wage adjustment even if some firms are not directly exposed to those shocks.

¹²We compute only the contribution of the indirect effects up to the tenth-order.

2.6 Calibration

We validate policy implications through numerical simulations; therefore, we need to specify parameter values for the above economy. Our baseline parameters are reported in Table A.8.

Household sector. Time is measured in quarters. The discount factor is chosen to yield a 4% of yearly real interest rate. This value is consistent with Aristizabal-Ramirez & Posso (2021), who report that the annual Colombian interbanking real interest rate in the fifth percentile during the period 2008-2018 is 4%. We calibrate parameter χ to match an unemployment rate of 10%, as in Aristizabal-Ramirez & Posso (2021). We set the Frisch elasticity parameter to 1. Parameter ω is calibrated to match the balance of payments account and is set to 0.975.

Production sector. We assume that the production technology follows a Cobb-Douglas form, with the elasticity of foreign inputs, $1 - \alpha$, set to 0.16. This parameter is taken from Mendoza & Yue (2012) to replicate elasticity of foreign inputs in production. We assume that probability of resetting price, $1 - \delta_p$, is set to match price adjustment on average in 3 quarters and is set to be 0.33. On the wage setting side, we assume that the Calvo parameter $1 - \delta_w$ is set so firms can reset wages every 4 quarters. Finally, parameters μ_w and μ_p are calibrated to generate gross markdown and markup values of 0.9 and 1.1, respectively.

Rest of the world. The parameters \bar{Z} and \bar{B}^* are calibrated to match aggregate revenues from commodities (petroleum products, coal, and basic metals) and net foreign assets as a share of GDP. We assume that the endowment of commodities is persistent, with a persistence parameter of 0.7. On the other hand, the parameter $(1 - \omega^*)C^*$ is calibrated to match the value of non-commodity exports. We assume that ν is equal to 4 borrowed from Galí & Monacelli (2005). Finally, the semi-elasticity of the foreign interest rate with respect to foreign debt is set to 0.1, a value chosen to ensure model closure. Parameter \bar{i}^* is calibrated to match the steady-state interest rate.

Monetary policy. We set the parameter ϕ_π to 1.5, a standard value in the literature. We assume that domestic monetary policy is inertial, with the persistence parameter ρ_M set to 0.5. Parameter \bar{i} is calibrated to match the steady-state interest rate.

2.7 Policy implications

In our baseline exercise, we examine the aggregate response of the entire economy to a negative commodity shock. Specifically, we assume a 10% decline in the dollar-valued endowment at period 0,

followed by a gradual recovery. We compare impulse response functions across two calibrations: *real rigidities* and *nominal rigidities*. The *real rigidities* calibration serves as the baseline, where nominal wages adjust over four quarters and strategic interactions in wage setting are present. Conversely, the *nominal rigidities* calibration excludes strategic interactions in wage setting and assumes that wages adjust over eight quarters. Additionally, we compare impulse responses under two distinct monetary regimes. In the first regime, the monetary authority follows a Taylor rule, adjusting nominal interest rates to stabilize inflation. In the second regime, the authority engages in foreign exchange interventions to stabilize the nominal exchange rate, prioritizing exchange rate stability over inflation control.

The evolution of the economy under the first monetary regime is depicted in Figure B.10. The negative commodity shock leads to a significant and prolonged exchange rate depreciation. Quantitatively, the exchange rate depreciates similarly across both calibrations: *real rigidities* and *nominal rigidities*. However, the dynamics of real wages and employment in response to the shock differ notably between the two. Under *real rigidities*, the adjustment of real wages and employment is sluggish. This sluggishness is primarily driven by the slow adjustment of nominal wages, which is a consequence of strategic interactions in wage setting. In contrast, under *nominal rigidities*, wages adjust more quickly. The dynamics of aggregate nominal wages illustrate that they are less responsive in the *real rigidities* calibration compared to the *nominal rigidities* calibration. The real exchange rate depreciation puts downward pressure on the value-added generated by a marginal worker. Strategic interactions make firms less inclined to deviate from the wage-setting behavior of competitors affected by the shock.

In contrast, Figure B.11 illustrates the economy's evolution under the second monetary regime. An examination of the real exchange rate and inflation dynamics reveal that the monetary authority smooths nominal exchange rate fluctuation at the cost of deflation. This intervention also results in relatively stable real exchange rate dynamics compared to the first regime. While differences in dynamics between the *real rigidities* and *nominal rigidities* calibrations persist, exchange rate management helps prevent a decline in real wages and employment. This analysis builds on existing literature that supports government interventions in foreign exchange markets (e.g. Calvo & Reinhart (2002)). It demonstrates that foreign exchange interventions can mitigate the cost of exchange rate depreciations.

Concluding remarks

In this paper, we examine the role of strategic interactions between firms in wage setting and their implications for aggregate wage adjustment. We identify the strategic interactions through a network-based approach leveraging recent developments in the estimation of peer effects and granular administrative employee-employer data from Colombia. First, instead of merely regressing firm's wage adjustment on average labor market wage adjustment, we exploit the network structure of competition with firms competing with a set of firms in a given market. The recent developments in the estimation of peer effects allow us to take care of the simultaneity problem and unobservable correlated effects. Second, we build the network structure of firms' competition using job-to-job flows from the administrative employee-employer data. Our findings reveal that external shocks can significantly influence firms' wage adjustments, even when firms are not directly exposed to these shocks. First, firms adjust their wages by approximately 1% in response to a 10% change in the wages of their competitors, controlling for external shocks. Second, firms respond to external shocks faced by their competitors, lowering wages by 6% in response to a 10% decline in the value added by their competing firms.

The strategic interactions contribute to sluggish aggregate wage adjustment following negative external shocks. Aggregate wages in labor markets affected by external shocks could be approximately 2 percentage points lower over a two-year horizon due to the strategic interactions in wage setting and network effects. Furthermore, our firm-to-firm network estimation shows that firms down the job ladder compete with those above. Thus, external shocks originating at the top to propagate throughout the entire job ladder. The estimated network formation model implies that firms at the top of the wage job ladder and larger employers receive greater influence in the network structure. Consequently, if these dominant firms are significantly impacted by external shocks, their wage-setting responses could contribute disproportionately to sluggish aggregate wage adjustments. This phenomenon highlights the critical role of the largest employers in shaping local labor market dynamics and the propagation of external shocks through the network. We also find that firm networks exhibit a substantial degree of homophily in terms of the quality of workers and their observable characteristics.

Using our general equilibrium model with oligopsonistic labor markets, we show that external shocks - such as those stemming from commodity price changes or capital flows - can result in sluggish aggregate wage adjustment. This outcome is primarily driven by the strategic interactions in wage setting that we model following Kimball (1995). Furthermore, we find that domestic mone-

tary policy, particularly foreign exchange interventions, is more effective than inflation targeting in addressing these dynamics. Foreign exchange interventions facilitate faster real wage adjustments, mitigating the sluggishness induced by strategic interactions in wage setting.

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A Tables

Table A.1. Summary statistics and AKM decomposition

	All (1)	All - AKM (2)	Above minimum wage (3)	Above minimum wage - AKM (4)
Sample size				
# of worker-years	72,447,390	71,444,873	61,955,631	61,447,455
# of workers	14,077,932	13,855,647	13,741,240	13,559,020
# of firms	829,037	657,836	727,749	599,458
# of MSA-firms		808,431	913,792	743,507
Summary statistics				
Mean of log wages	3.48	3.49	3.57	3.57
Variance of log wages	0.37	0.37	0.38	0.39
Share of variance of wages explained by each component				
MSA-firms		0.19		0.17
Workers		0.60		0.62
Xb		0.11		0.11
Variance of components				
Variance of MSA-firm effects		0.14		0.13
Variance of worker effects		0.56		0.59
Variance of Xb		0.12		0.12
$2 \times \text{cov}(\text{MSA-firm, worker})$		0.10		0.09
$2 \times \text{cov}(\text{Xb, MSA-firm} + \text{worker})$		-0.026		-0.037
Overall fit of AKM decomposition				
Adj. R^2		0.8680		0.8647

Notes: The first column represents summary statistics for the full sample from 2009-2018. We keep only one job per worker per year chosen according to the number of days stayed at a given firm within a given year. The third column contains all observations excluding observations close to minimum wage within a given year. As a threshold, we drop all observations with a wage below minimum wage within a given year. In the second and fourth columns, AKM stands for the estimation procedure following Abowd et al. (1999) (AKM). Differently from the AKM, we define “firms” as MSA-firms – the same firms across different metropolitan statistical areas are considered to be different firms. The AKM sample contains all firms within the largest connected set within each metropolitan area. We also include MSA-specific time trends using MSA-year fixed effects; therefore, MSA-firm pay components are identified relative to a firm in the largest connected set in a given metropolitan area.

Table A.2. Metropolitan area coverage

Metropolitan Area	Frequency	Percent	Cumulative
Bogota	23,800,915	38.42	38.42
Medellin	9,389,587	15.16	53.57
Cali	5,943,712	9.59	63.16
Barranquilla	3,203,866	5.17	68.34
Cartagena	1,845,120	2.98	71.31
Bucaramanga	2,467,438	3.98	75.30
Cucuta	973,592	1.57	76.87
Pepeira	1,367,627	2.21	79.08
Sogamoso	140,556	0.23	79.30
Rionegro	140,556	0.67	79.97
Tunja	475,448	0.77	80.74
Armenia	601,098	0.97	81.71
Girardot	122,937	0.20	81.90
Villavicencio	1,056,725	1.71	83.69
Manizales	936,977	1.51	85.12
Pasto	559,133	0.90	86.03
Tulua	211,418	0.34	86.37
Ipiales	58,306	0.09	86.46
Duitama	134,352	0.22	86.68
Ibague	941,558	1.52	88.20
Santa Marta	707,476	1.14	89.34
Monteria	565,539	0.91	90.25
Valledupar	629,105	1.02	91.27
Buenaventura	205,266	0.33	91.60
Neiva	688,582	1.11	92.71
Palmira	342,976	0.55	93.26
Popayan	561,880	0.91	94.17
Sincelejo	307,737	0.50	94.67
Riohacha	259,555	0.42	95.09
Barrancabermeja	418,923	0.68	95.76
San Andres de Tumaco	63,227	0.10	95.86
Florencia	179,055	0.29	96.15
Apartado	210,090	0.34	96.49
Maicao	39,234	0.06	96.56
Turbo	43,381	0.07	96.63
Cartago	128,779	0.21	96.83
Yopal	443,664	0.72	97.55
Manague	33,443	0.05	97.60
Fusagasuga	81,792	0.13	97.74
Guadalajara de Buga	221,922	0.36	98.09
Quibdo	129,160	0.21	98.30
Others	1,051,840		100.00

Notes: PILA coverage as of 2009-2018. The table includes one observation According to the DANE classification there are 62 metropolitan areas. The list of not included metropolitan areas is Lorica, Pitalito, Cienaga, Caucasia, Ocana, Aguachica, Cerete, Santander de Quilichao, Arauca, Espinal, La Dorada, Montelibano, El Carmen de Bolivar, Chigorodo, Acacias, Chiquinquirá, Corozal, Fundacion, Granada, Pamplona. The full PILA coverage is 113,489,936 firm-worker-year observations. We exclude 6,056,150 firm-worker-year observations not included in one of the metropolitan areas.

Table A.3. Industry coverage

Industry	Frequency	Percent	Cumulative
Agriculture, hunting and related service activities	2,423,216	3.91	3.91
Forestry, logging and related service activities	77,660	0.13	4.04
Fishing, aquaculture and service activities incidental to fishing	18,104	0.03	4.07
Mining of coal and lignite; extraction of peat	164,356	0.27	4.33
Extraction of crude petroleum and natural gas; service activities incidental to oil and gas extraction, excluding surveying	197,812	0.32	4.65
Mining of uranium and thorium ores	3,848	0.01	4.66
Mining of metal ores	27,905	0.05	4.70
Other mining and quarrying	38,875	0.06	4.76
Manufacture of food products and beverages	1,395,208	2.25	7.02
Manufacture of tobacco products	14,353	0.02	7.04
Manufacture of textiles	317,260	0.51	7.55
Manufacture of wearing apparel; dressing and dyeing of fur	824,357	1.33	8.88
Tanning and dressing of leather; manufacture of luggage, handbags, saddlery, harness and footwear	140,774	0.23	9.11
Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials	111,806	0.18	9.29
Manufacture of paper and paper products	158,266	0.26	9.55
Publishing, printing and reproduction of recorded media	461,901	0.75	10.29
Manufacture of coke, refined petroleum products and nuclear fuel	25,169	0.04	10.33
Manufacture of chemicals and chemical products	680,322	1.10	11.43
Manufacture of rubber and plastics products	389,355	0.63	12.06
Manufacture of other non-metallic mineral products	263,823	0.43	12.48
Manufacture of basic metals	149,913	0.24	12.73
Manufacture of fabricated metal products, except machinery and equipment	389,609	0.63	13.35
Manufacture of machinery and equipment n.e.c.	254,140	0.41	13.76
Manufacture of office, accounting and computing machinery	3,185	0.01	13.77
Manufacture of electrical machinery and apparatus n.e.c.	183,477	0.30	14.07
Manufacture of radio, television and communication equipment and apparatus	10,508	0.02	14.08
Manufacture of medical, precision and optical instruments, watches and clocks	120,976	0.20	14.28
Manufacture of motor vehicles, trailers and semi-trailers	165,759	0.27	14.55
Manufacture of other transport equipment	77,283	0.12	14.67
Manufacture of furniture; manufacturing n.e.c.	314,191	0.51	15.18
Recycling	30,393	0.05	15.23
Electricity, gas, steam and hot water supply	144,705	0.23	15.46
Collection, purification and distribution of water	172,874	0.28	15.74
Construction	4,912,101	7.93	23.67
Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of automotive fuel	1,061,692	1.71	25.38

Notes: PILA coverage as of 2009-2018. See the next page for a full list of industries and a full legend.

Table A.4. Industry coverage (continued)

Industry	Frequency	Percent	Cumulative
Wholesale trade and commission trade, except of motor vehicles and motorcycles	2,932,232	4.73	30.11
Retail trade, except of motor vehicles and motorcycles; repair of personal and household goods	2,881,513	4.65	34.77
Hotels and restaurants	1,194,675	1.93	36.69
Land transport; transport via pipelines	1,857,053	3.00	39.69
Water transport	47,488	0.08	39.77
Air transport	111,235	0.18	39.95
Supporting and auxiliary transport activities; activities of travel agencies	567,328	0.92	40.86
Post and telecommunications	928,019	1.50	42.36
Financial intermediation, except insurance and pension funding	1,109,223	1.79	44.15
Insurance and pension funding, except compulsory social security	1,523,719	2.46	46.61
Activities auxiliary to financial intermediation	793,116	1.28	47.89
Real estate activities	596,691	0.96	48.85
Renting of machinery and equipment without operator and of personal and household goods	177,957	0.29	49.14
Computer and related activities	725,435	1.17	50.31
Research and development	256,154	0.41	50.73
Other business activities	17,658,971	28.50	79.23
Public administration and defense; compulsory social security	2,570,949	4.15	83.38
Education	2,554,584	4.12	87.50
Health and social work	2,981,900	4.81	92.31
Sewage and refuse disposal, sanitation and similar activities	109,789	0.18	92.49
Activities of membership organizations n.e.c.	2,458,075	3.97	96.46
Recreational, cultural and sporting activities	746,400	1.20	97.66
Other service activities	1,153,631	1.86	99.52
Activities of private households as employers of domestic staff	128,179	0.21	99.73
Undifferentiated goods-producing activities of private households for own use	7,162	0.01	99.74
Undifferentiated service-producing activities of private households for own use	158,977	0.26	100.00

Notes: PILA coverage as of 2009-2018. We use the International Standard Industrial Classification Revision 3.1 of industries at 2-digit level. The full PILA coverage is 113,489,936 firm-worker-year observations. We exclude 4,022,006 firm-worker-year observations with missing industry information.

Table A.5. Additional coverage

	Frequency	Percent	Cumulative
<u>Year</u>			
2009	4,803,671	7.75	7.75
2010	4,996,790	8.07	15.82
2011	5,316,643	8.58	24.40
2012	5,906,325	9.53	33.93
2013	6,274,129	10.13	44.06
2014	6,943,173	11.21	55.27
2015	7,258,075	11.71	66.98
2016	6,941,343	11.20	78.19
2017	6,878,452	11.10	89.29
2018	6,637,030	10.71	100.00
<u>Gender</u>			
Female	25,647,432	41.40	41.40
Male	36,307,214	58.60	100.00
Not reported	985	0.00	100.00
<u>Skills</u>			
Low-skill	30,723,625	49.59	49.59
High-skill	30,723,830	49.59	99.18
Not defined	508,176	0.82	100.00

Notes: PILA coverage as of 2009-2018. The sample includes only one job per worker per year chosen according to the number of days stayed at a given firm within a given year. We exclude all observations outside of the 62 metropolitan areas, observations with missing industry data, and with missing wages.

Table A.6. Network estimation

	Bogota Metropolitan Area			Medellin Metropolitan Area			Cali Metropolitan Area		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	-16.4703 (0.0046)	-16.5131 (0.0047)	-16.4883 (0.0046)	-14.6231 (0.0064)	-14.6368 (0.0065)	-14.6326 (0.0064)	-13.8307 (0.0085)	-13.8551 (0.0086)	-13.8516 (0.0085)
1-digit industry		0.2386 (0.0023)			0.0823 (0.0041)			0.1293 (0.0054)	
2-digit industry			0.3070 (0.0025)			0.1290 (0.0046)			0.2102 (0.0061)
$\log(\text{empl}_i)$	1.0436 (0.0005)	1.0421 (0.0005)	1.0397 (0.0005)	1.0216 (0.0008)	1.0211 (0.0008)	1.0203 (0.0008)	1.0429 (0.0011)	1.0426 (0.0011)	1.0414 (0.0011)
$\log(\text{empl}_j)$	1.1482 (0.0005)	1.1468 (0.0005)	1.1444 (0.0005)	1.0487 (0.0008)	1.048 (0.0008)	1.0471 (0.0008)	1.0233 (0.0011)	1.023 (0.0011)	1.0219 (0.0011)
firm pay component	0.4013 (0.0035)	0.4069 (0.0035)	0.4064 (0.0035)	0.4806 (0.0064)	0.4823 (0.0064)	0.4826 (0.0064)	0.5362 (0.0076)	0.5393 (0.0077)	0.5402 (0.0077)
worker pay component	-0.1778 (0.003)	-0.1552 (0.003)	-0.1421 (0.003)	-0.4455 (0.0059)	-0.4360 (0.0058)	-0.4281 (0.0059)	-0.5202 (0.0084)	-0.5008 (0.0084)	-0.4865 (0.0084)
share of females	-0.5220 (0.0053)	-0.4863 (0.0053)	-0.4905 (0.0053)	-0.0603 (0.0075)	-0.0483 (0.0075)	-0.0448 (0.0075)	-0.3516 (0.0104)	-0.3362 (0.0104)	-0.3330 (0.0104)
# of firm pairs	40,000-by-40,000			26,055-by-26,055			20,000-by-20,000		
Employment covered	1.696M out of 1.848M			0.697M out of 0.724M			0.440M out of 0.446M		
Flow of workers covered	1.239M out of 1.376M			0.439M out of 0.462M			0.270M out of 0.274M		

Notes: Standard errors are in parentheses. We estimate the flow of workers between firm i and j , using the pseudo-maximum likelihood approach motivated by Silva & Tenreyro (2006). We report only estimates for the six largest metropolitan areas in Colombia: Bogota, Medellin, Cali, Barranquilla, Cartagena, and Bucaramanga. For each metropolitan area, we estimate three versions of the network formation. Each version contains a constant, logarithms of employment by firm i and j , the difference in firm pay components estimated following Abowd et al. (1999), and the absolute value of differences in average worker pay components and share of females in a firm. The three versions differ in including 1-digit or 2-digit industry indicators if firms are in the same industry. (continued in the next page)

Table A.7. Network estimation (continued)

	Barranquilla Metropolitan Area			Cartagena Metropolitan Area			Bucaramanga Metropolitan Area		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	-13.4395 (0.0113)	-13.526 (0.0114)	-13.4926 (0.0113)	-12.5453 (0.0125)	-12.5895 (0.0126)	-12.5799 (0.0126)	-12.8291 (0.0118)	-12.8925 (0.0119)	-12.8787 (0.0119)
1-digit industry		0.4603 (0.0058)			0.2166 (0.0079)			0.2751 (0.0076)	
2-digit industry			0.5376 (0.0060)			0.2554 (0.0084)			0.3206 (0.0083)
log (empl _i)	1.1233 (0.0015)	1.1152 (0.0015)	1.1104 (0.0015)	1.0485 (0.0018)	1.0464 (0.0018)	1.0448 (0.0018)	1.0438 (0.0017)	1.0436 (0.0017)	1.0421 (0.0016)
log (empl _j)	1.0744 (0.0015)	1.0664 (0.0015)	1.0616 (0.0015)	1.0365 (0.0017)	1.0344 (0.0017)	1.0329 (0.0017)	0.9938 (0.0017)	0.9936 (0.0016)	0.9921 (0.0016)
firm pay component	0.494 (0.0099)	0.5019 (0.0010)	0.5012 (0.0010)	0.3895 (0.0094)	0.3935 (0.0094)	0.3935 (0.0017)	0.2687 (0.0104)	0.2704 (0.0104)	0.2702 (0.0104)
worker pay component	-0.9516 (0.0108)	-0.8107 (0.0109)	-0.7785 (0.0109)	-0.5803 (0.0140)	-0.5476 (0.0140)	-0.5389 (0.0140)	-0.3362 (0.0125)	-0.3082 (0.0125)	-0.3039 (0.0125)
share of females	-0.2049 (0.0128)	-0.1465 (0.0128)	-0.1415 (0.0128)	-0.4031 (0.0148)	-0.3605 (0.0148)	-0.3555 (0.0148)	-0.1326 (0.0135)	-0.0903 (0.0135)	-0.0863 (0.0135)
# of firm pairs	15,397-by-15,397			11,138-by-11,138			20,000-by-20,000		
Employment covered	241,359			153,700			201,603		
Flow of workers covered	155,690			96,738			116,195		

Notes: Due to potential high dimensionality, we estimate the workers only among the largest firms in a given metropolitan area. For example, the estimation of worker flows between 100,000 firms would require 80 GB of RAM for one variable - with 1 endogenous variable and 6 exogenous variables requiring at least 560 GB of RAM not including temporary storage between operations. Therefore, we estimate the worker flows among the 40,000 largest firms in a given metropolitan area or the largest firms covering at least 95% of worker flows between each other whichever requirement meets first.

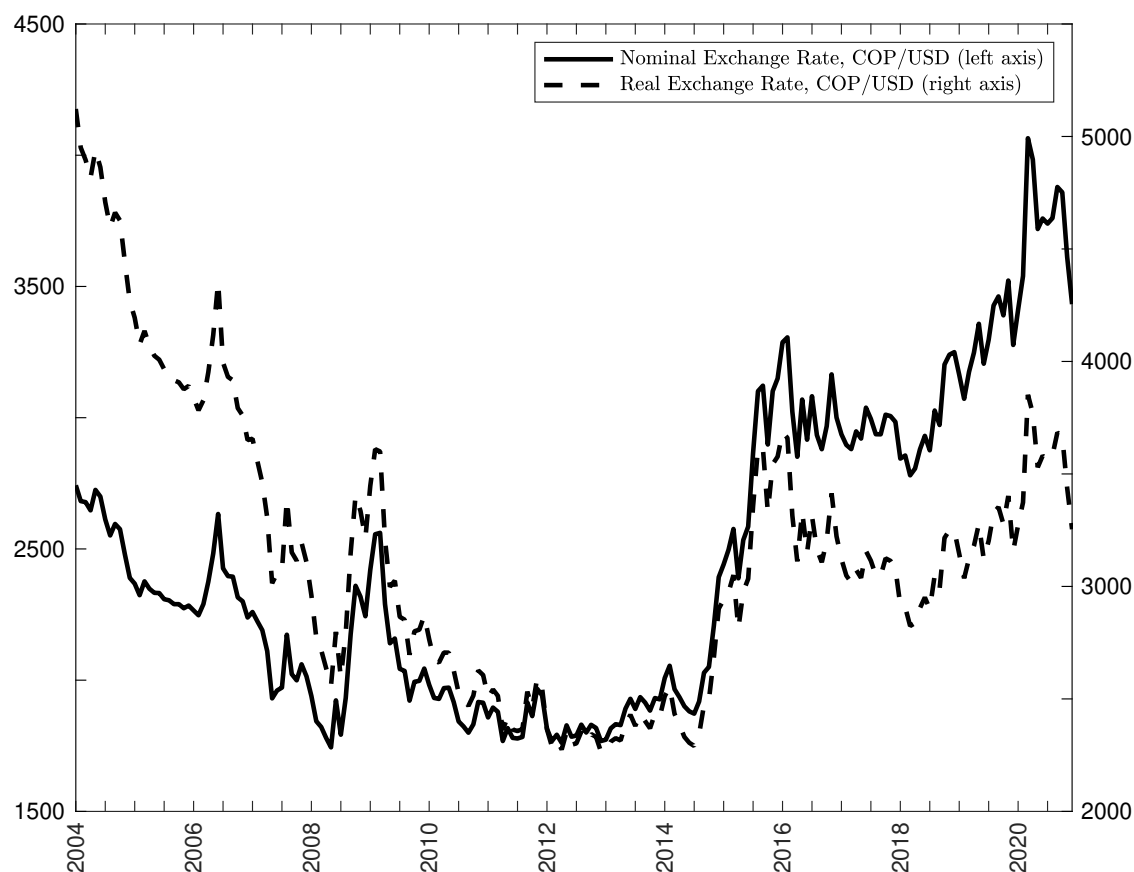
Table A.8. Calibration

Parameter	Explanation	Value	Target / Source
β	Discount factor	0.99	Annual interest rate $r = 4.0\%$
σ_n	Frisch elasticity	1.00	Linear-in-leisure utility function
χ	Disutility of labor	1.17	10% unemployment rate
ω	Share of Home goods	0.975	Balance of payment condition
$1 - \alpha$	Curvature in foreign inputs	0.16	Mendoza & Yue (2012)
\bar{Z}	Commodity endowment	0.096	10 % of GDP
\bar{B}^*	Foreign debt	0.298	31 % of GDP
ζ	Kimball super-elasticity	-10.00	XXX
μ_w	Wage gross markdown	0.9	10 % mark-down
μ_p	Price gross markup	1.1	10 % mark-up
$1 - \delta_w$	Probability of resetting wage	0.125	4 quarters to adjust
$1 - \delta_p$	Probability of resetting price	0.125	3 quarters to adjust
$(1 - \omega^*) C^*$	Foreign demand	0.5	Non-commodity export value
ν	Foreign demand elasticity	4	Galí & Monacelli (2005)
ψ	Foreign interest rate elasticity	0.01	Value enough to close model
ϕ_π	Reaction interest rate to inflation	1.5	Standard value
ρ_M	Persistence of MP shocks	0.5	Standard value
ρ_Z	Persistence of commodity shocks	0.7	Gopinath et al. (2020)
\bar{i}^*	Foreign interest rate (s.s. value)	1%	Annual interest rate $r = 4.0\%$
\bar{i}	Domestic interest rate (s.s. value)	1%	Annual interest rate $r = 4.0\%$

Notes: In our calibration, time period is a quarter.

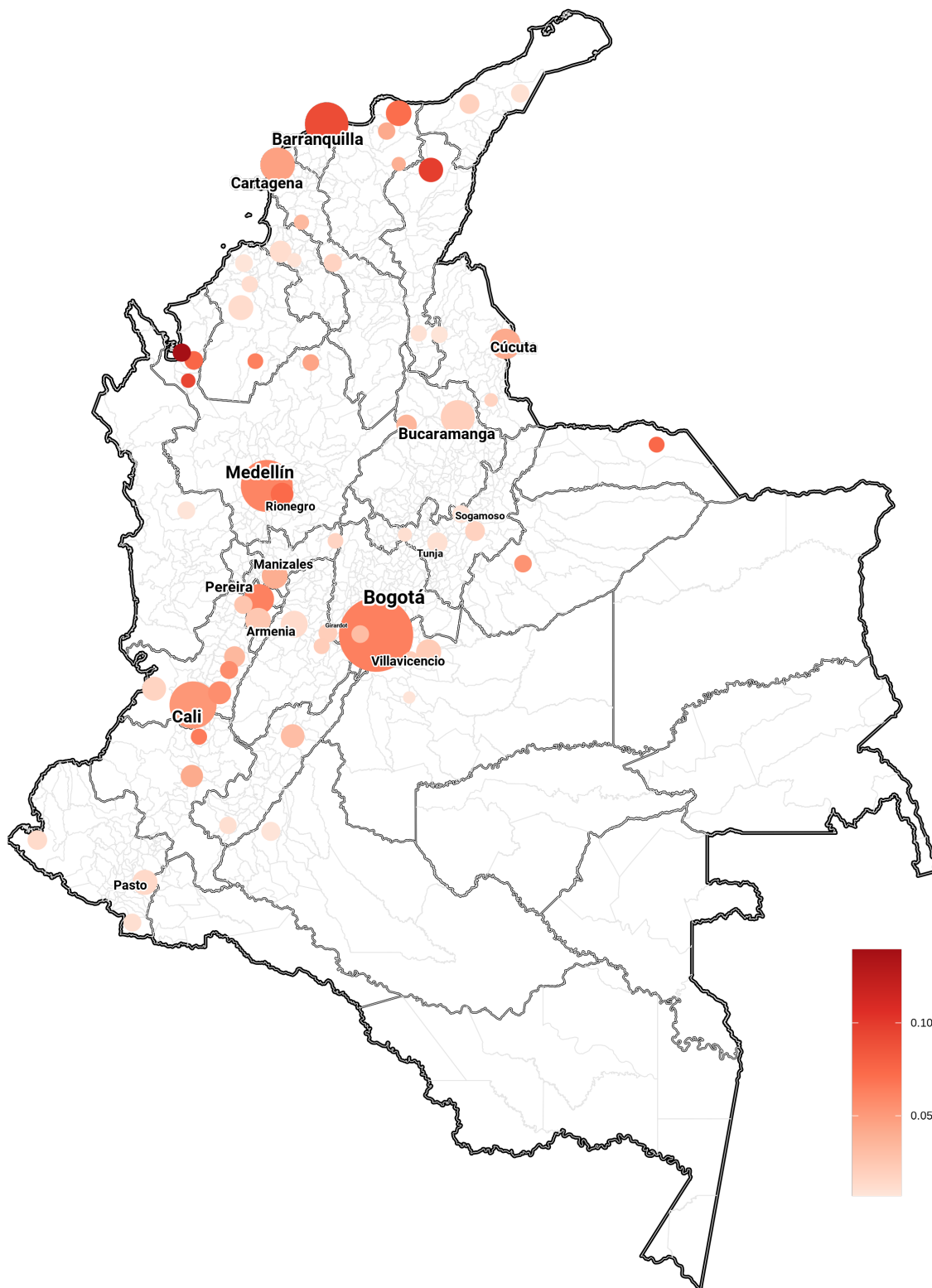
B Figures

Figure B.1. Market Exchange Rate of Colombian Peso



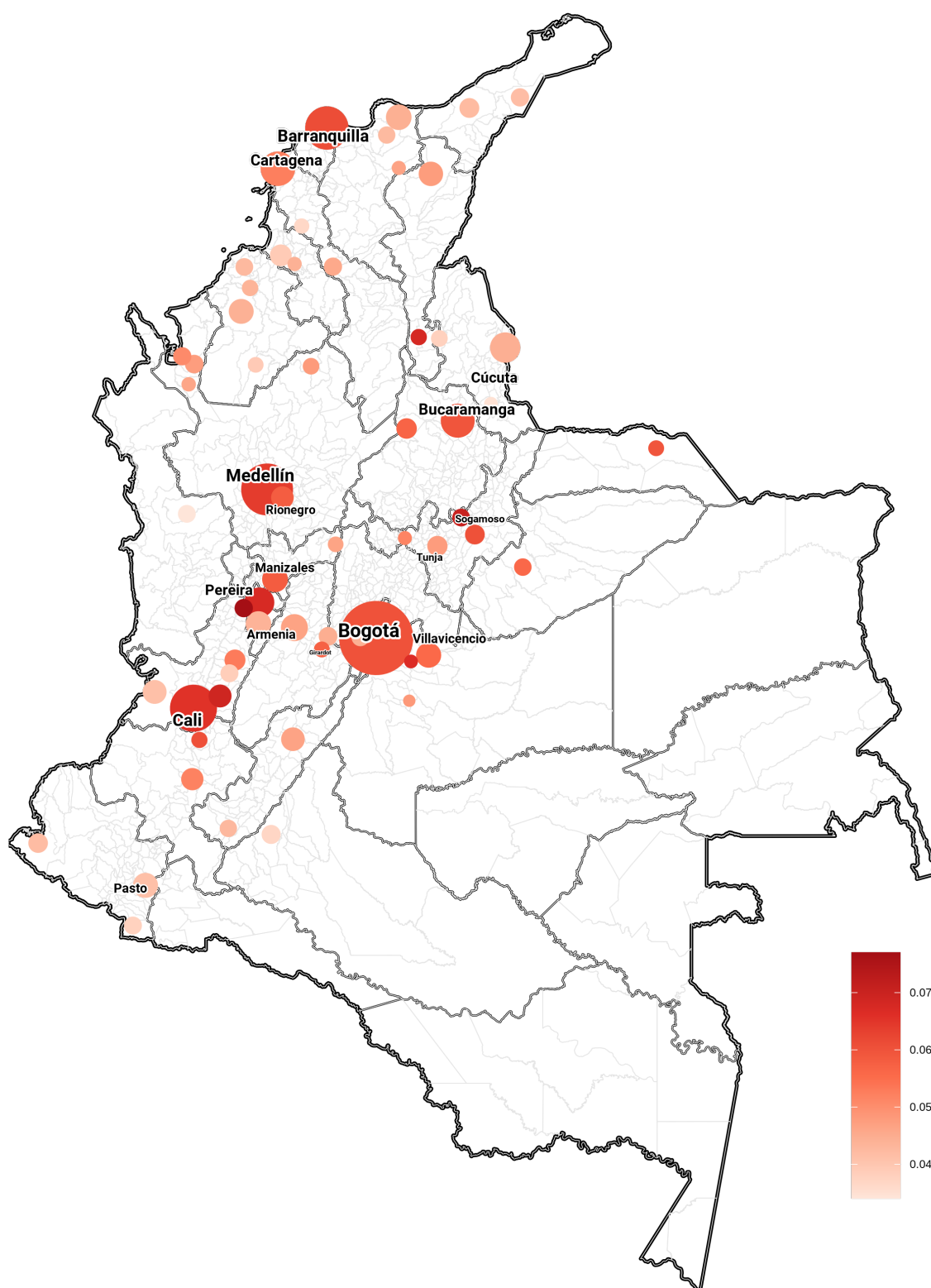
Notes: The source of data are the Central Bank of Colombia (*Banco de la Republica*) and the National Administrative Department of Statistics (*Departamento Administrativo Nacional de Estadística*). The exchange rate is in PESO per USD. The nominal exchange rate is deflated with Consumer Price Index. December 2018 is a base period. Data is represented on a monthly basis and as of end of a month.

Figure B.2. MSA Export Exposure



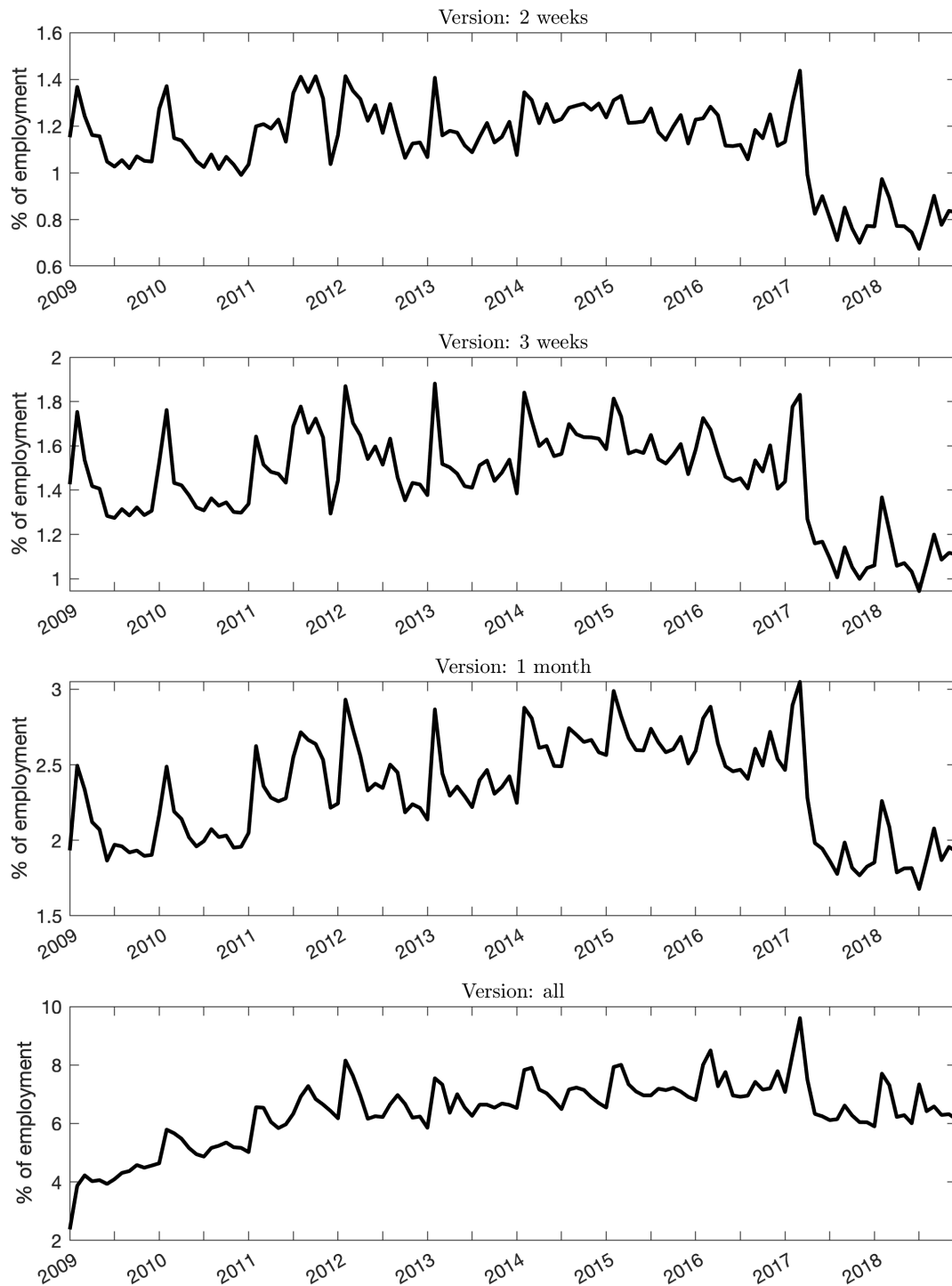
Notes: The figure shows the exposure of metropolitan areas (a total of 62 MSAs) to export, measured as a share of sales abroad. For each MSA, the exposure is calculated using employment weights at the 2-digit industry level.

Figure B.3. MSA Import Exposure



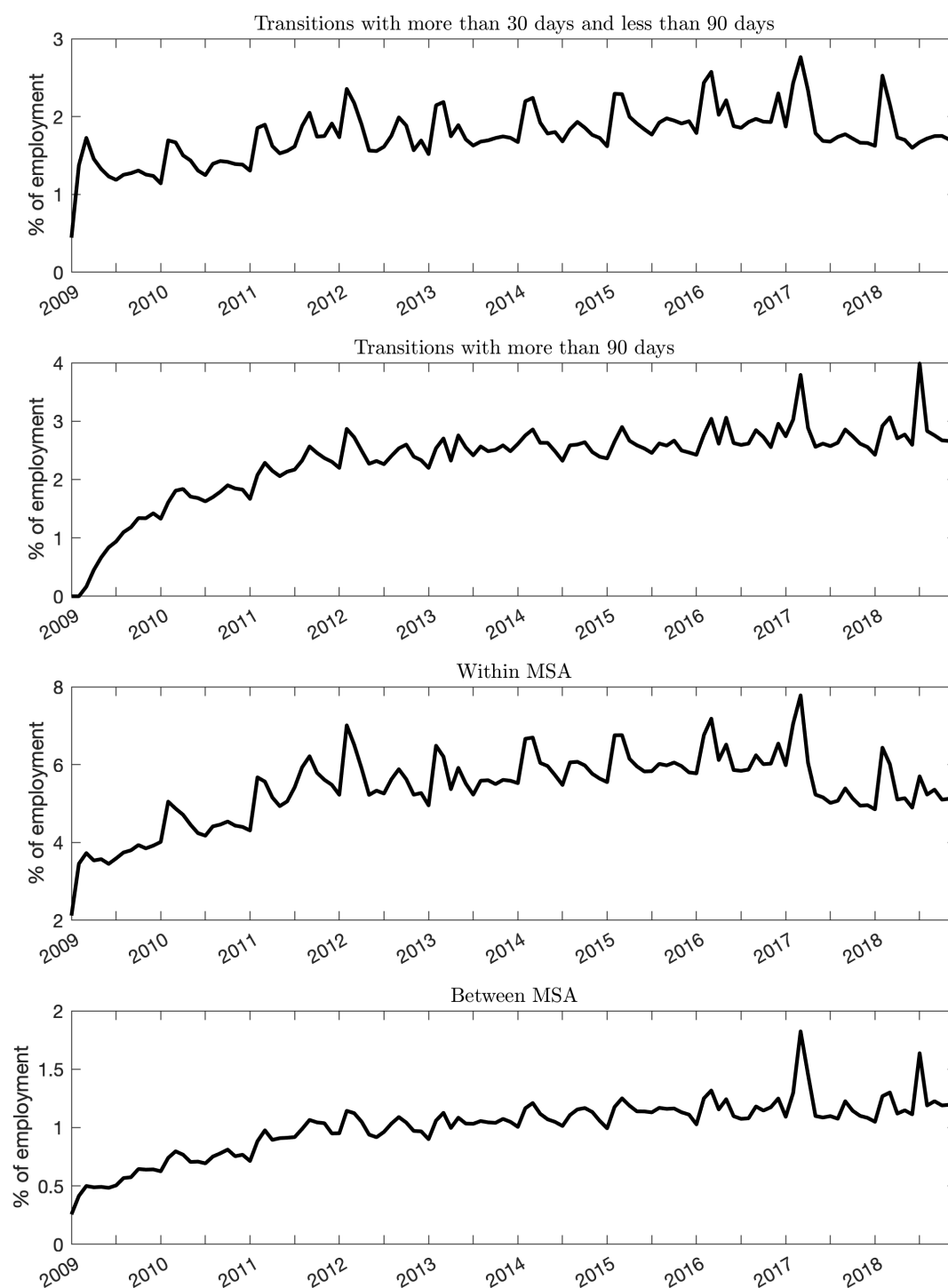
Notes: The figure shows the exposure of metropolitan areas (a total of 62 MSAs) to usage of foreign intermediate inputs, measured as a share of inputs imported from abroad. For each MSA, the exposure is calculated using employment weights at the 2-digit industry level.

Figure B.4. Employment Transition Rates



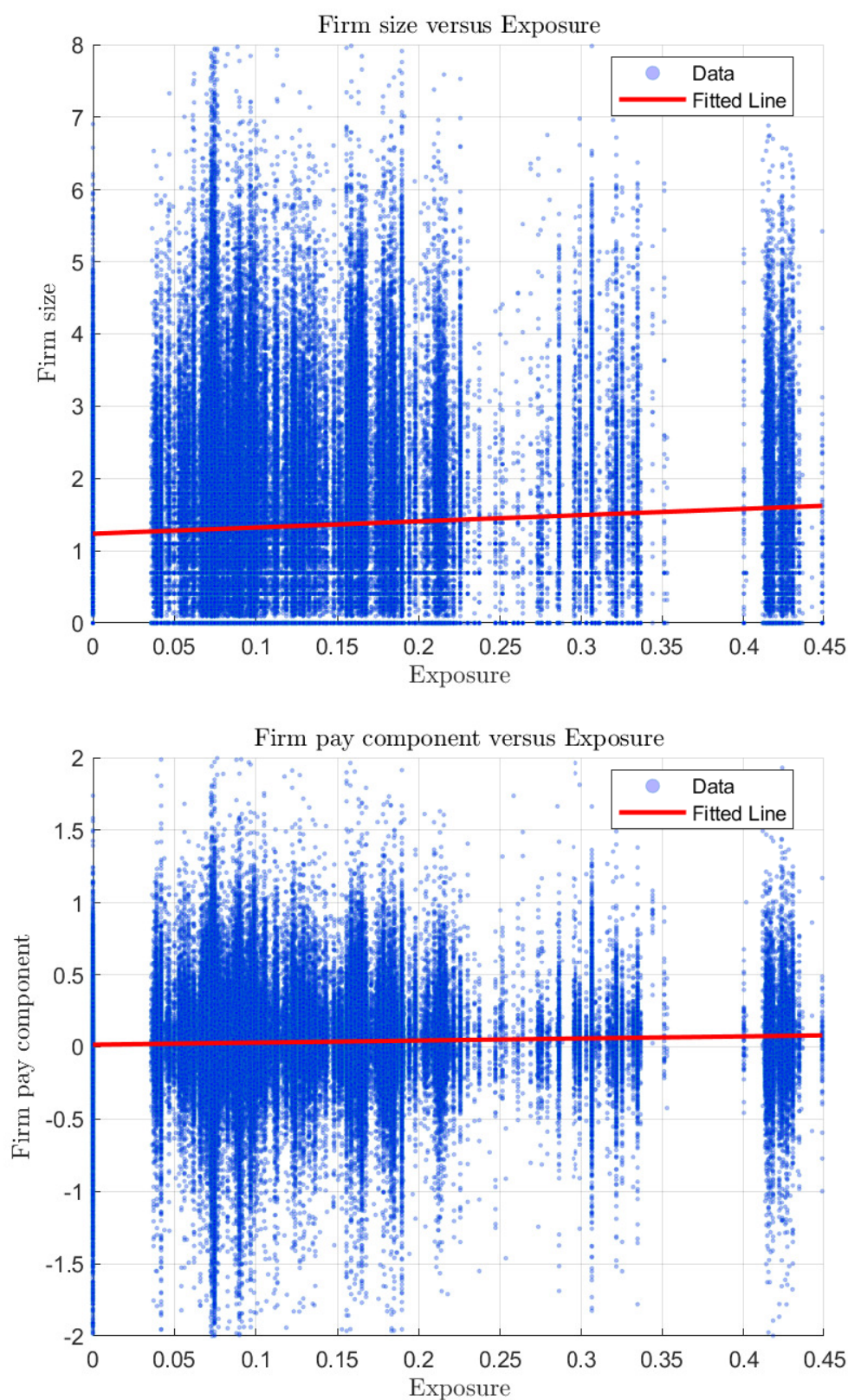
Notes: The figure plots transition probabilities defined as the share of workers experiencing job-to-job transitions within 2 weeks, 3 weeks, and 1 month relative to total number of workers. The last sub-figure plots share of all workers transitioning to from job to job with unrestricted duration of “unemployment” spells.

Figure B.5. Employment Transition Rates (continued)



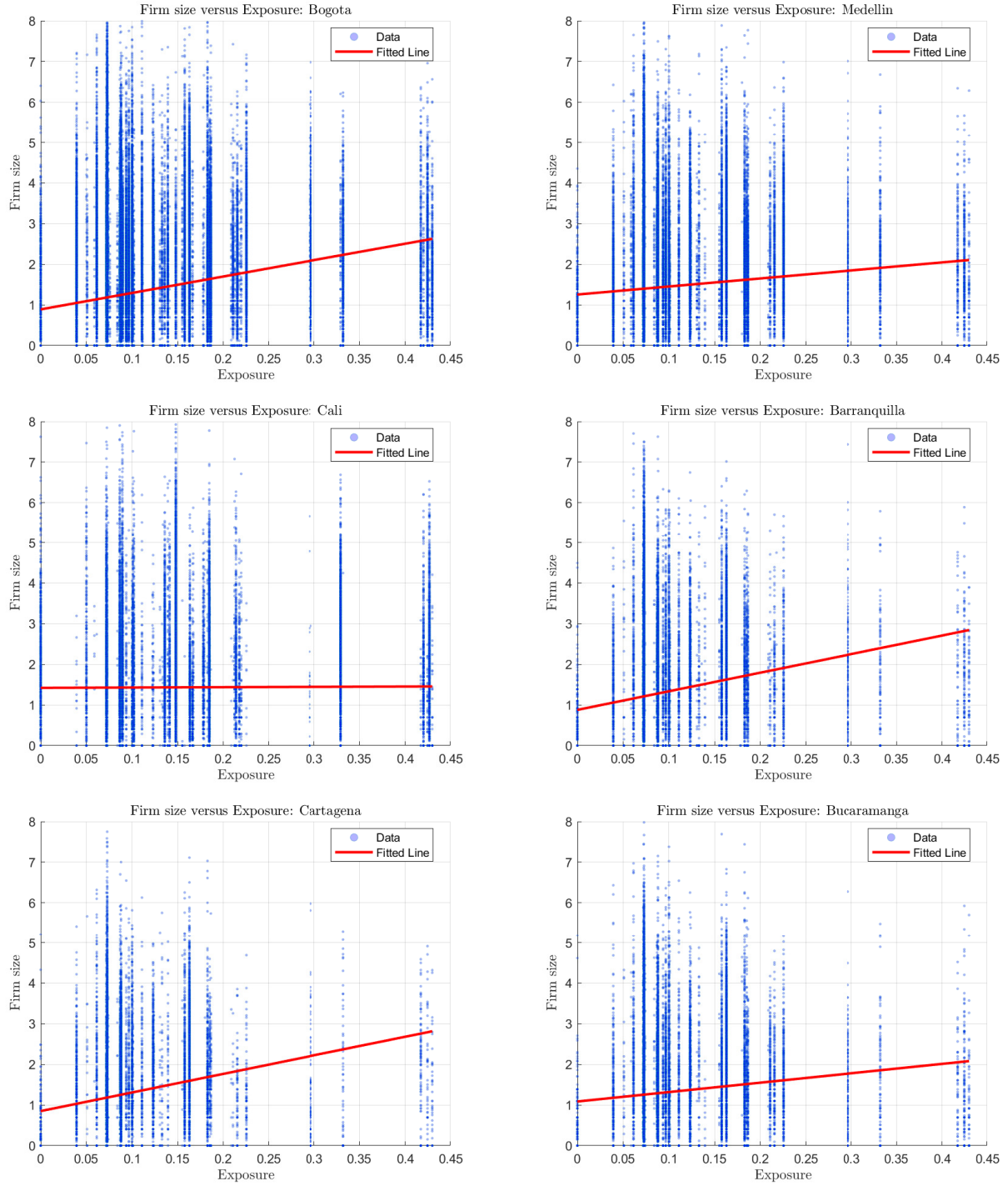
Notes: The first two sub-figures plot transition probabilities for workers with transitions (i) more than 30 days and less than 90 days, and (ii) more than 90 days. The third sub-figure plots transition probabilities that happen within a given metropolitan area. The last sub-figure plots transition rates across across metropolitan areas.

Figure B.6. Job Ladders and Exposure: Entire economy



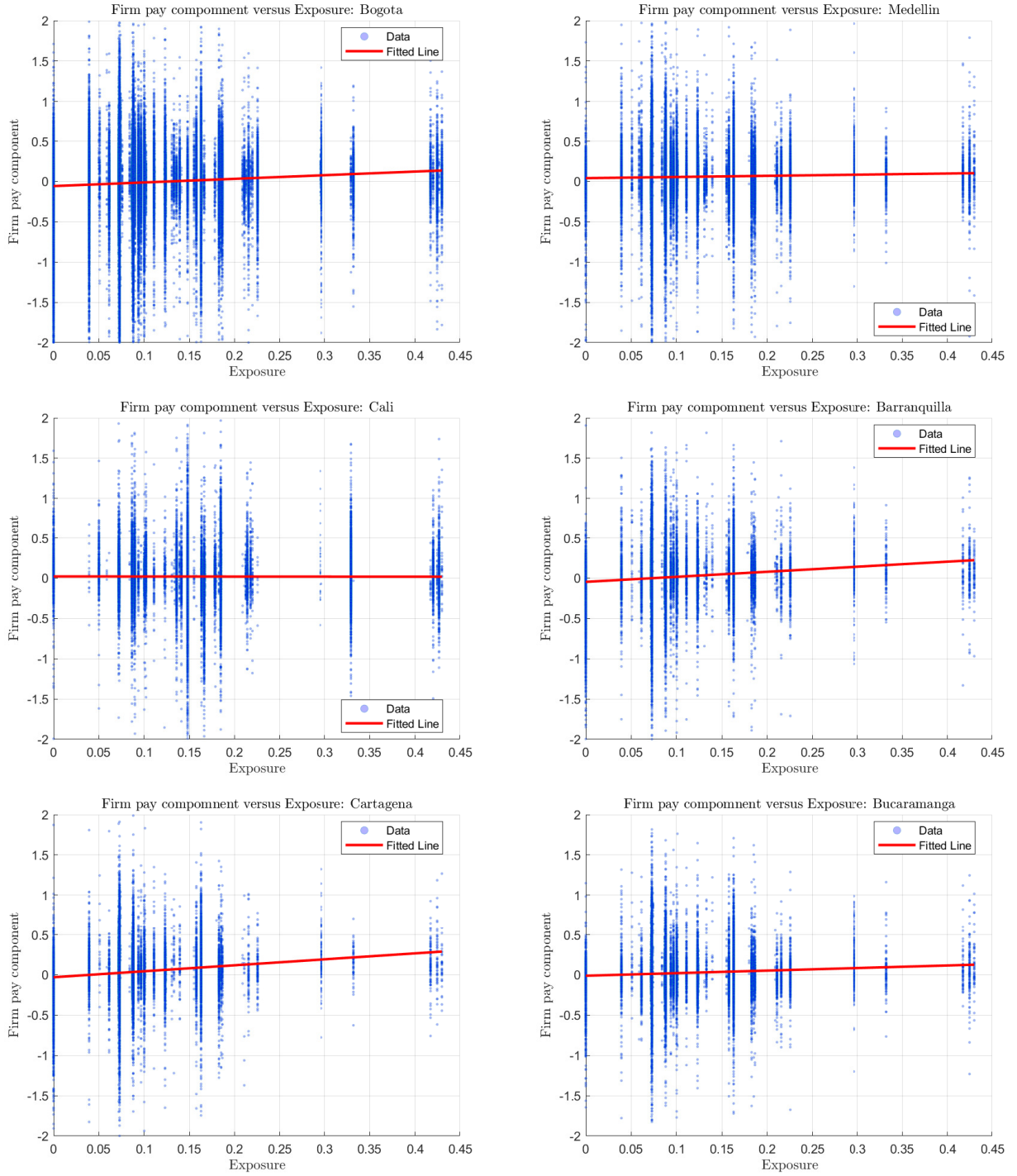
Notes: The figure illustrates the relationship between job ladders and firms' exposure to external shocks across the entire economy. The top panel presents this relationship for the firm size job ladder, with the fitted line in red showing a significant upward trend, having a slope of 0.86 (0.015). The bottom panel shows the relationship for the firm pay job ladder, with the fitted line in red also increasing significantly, with a slope of 0.144 (0.004), with standard errors in parentheses.

Figure B.7. Size Job Ladders and Exposure: MSA level



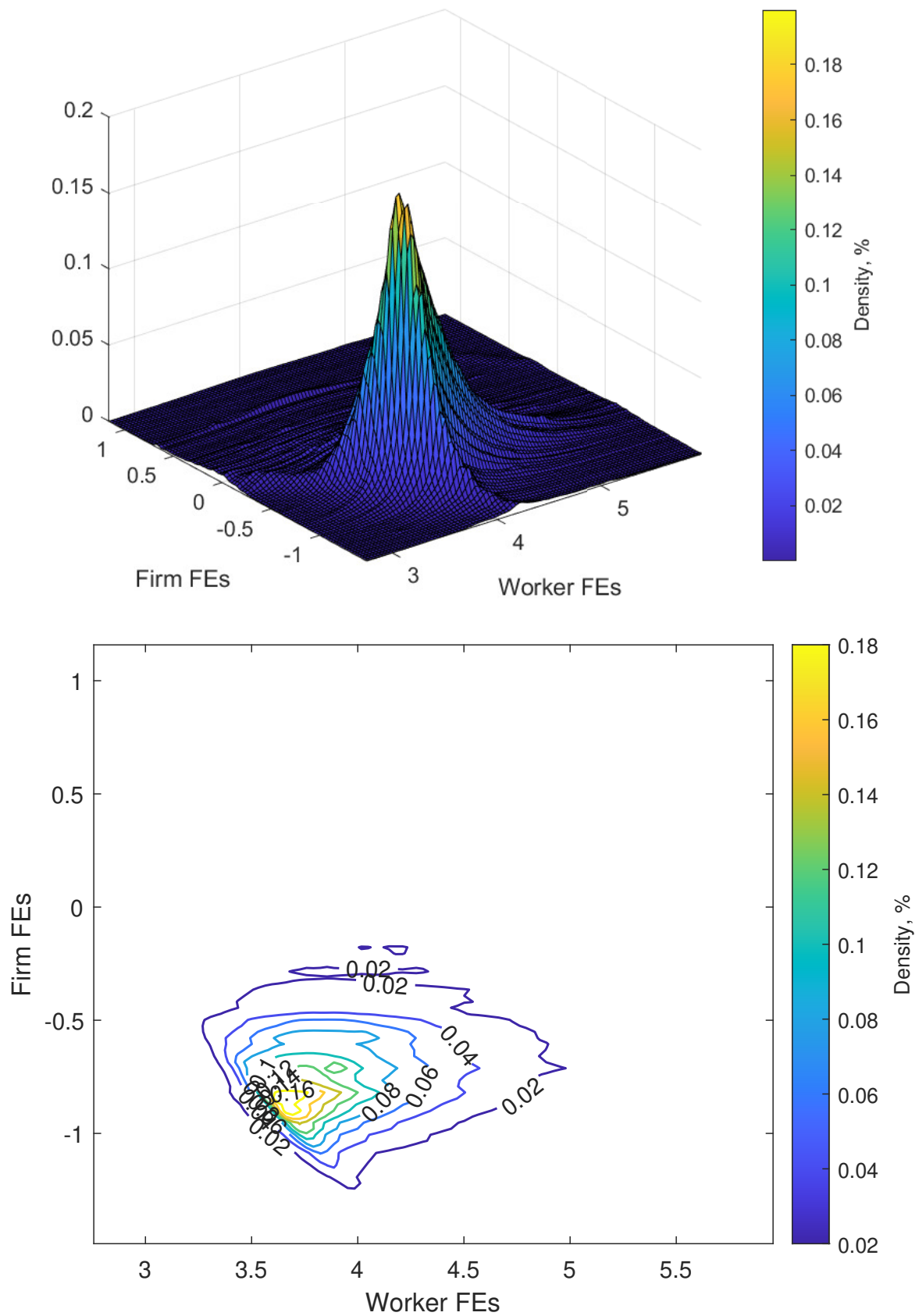
Notes: The figure illustrates the relationship between firm size job ladders and firms' exposure to external shocks across the six largest metropolitan areas: Bogota, Medellin, Cali, Barranquilla, Cartagena, and Bucaramanga. The fitted lines in red show significant positive slopes for all metropolitan areas except Cali. The slopes are: 4.04 (0.037) for Bogota, 1.98 (0.066) for Medellin, 0.08 (0.056) for Cali, 4.58 (0.099) for Barranquilla, 4.58 (0.119) for Cartagena, and 2.32 (0.104) for Bucaramanga, with standard errors in parentheses.

Figure B.8. Firm Pay Job Ladders and Exposure: MSA level



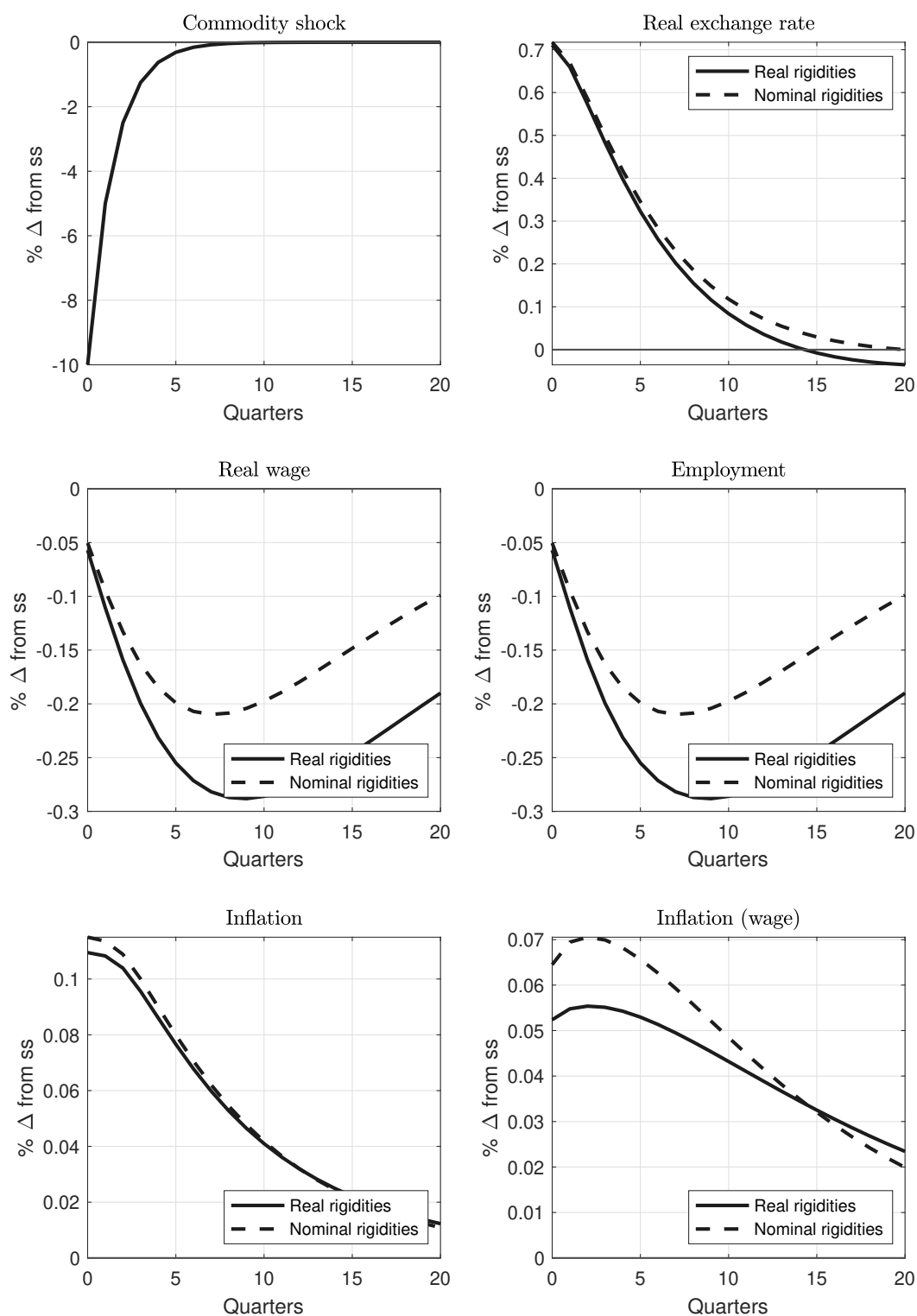
Notes: The figure illustrates the relationship between firm pay job ladders and firms' exposure to external shocks across the six largest metropolitan areas: Bogota, Medellin, Cali, Barranquilla, Cartagena, and Bucaramanga. The fitted lines in red show significant positive slopes for all metropolitan areas except Cali. The slopes are: 0.45 (0.011) for Bogota, 0.14 (0.016) for Medellin, -0.008 (0.013) for Cali, 0.63 (0.026) for Barranquilla, 0.74 (0.033) for Cartagena, and 0.32 (0.025) for Bucaramanga, with standard errors in parentheses.

Figure B.9. Distribution of firm and worker pay components



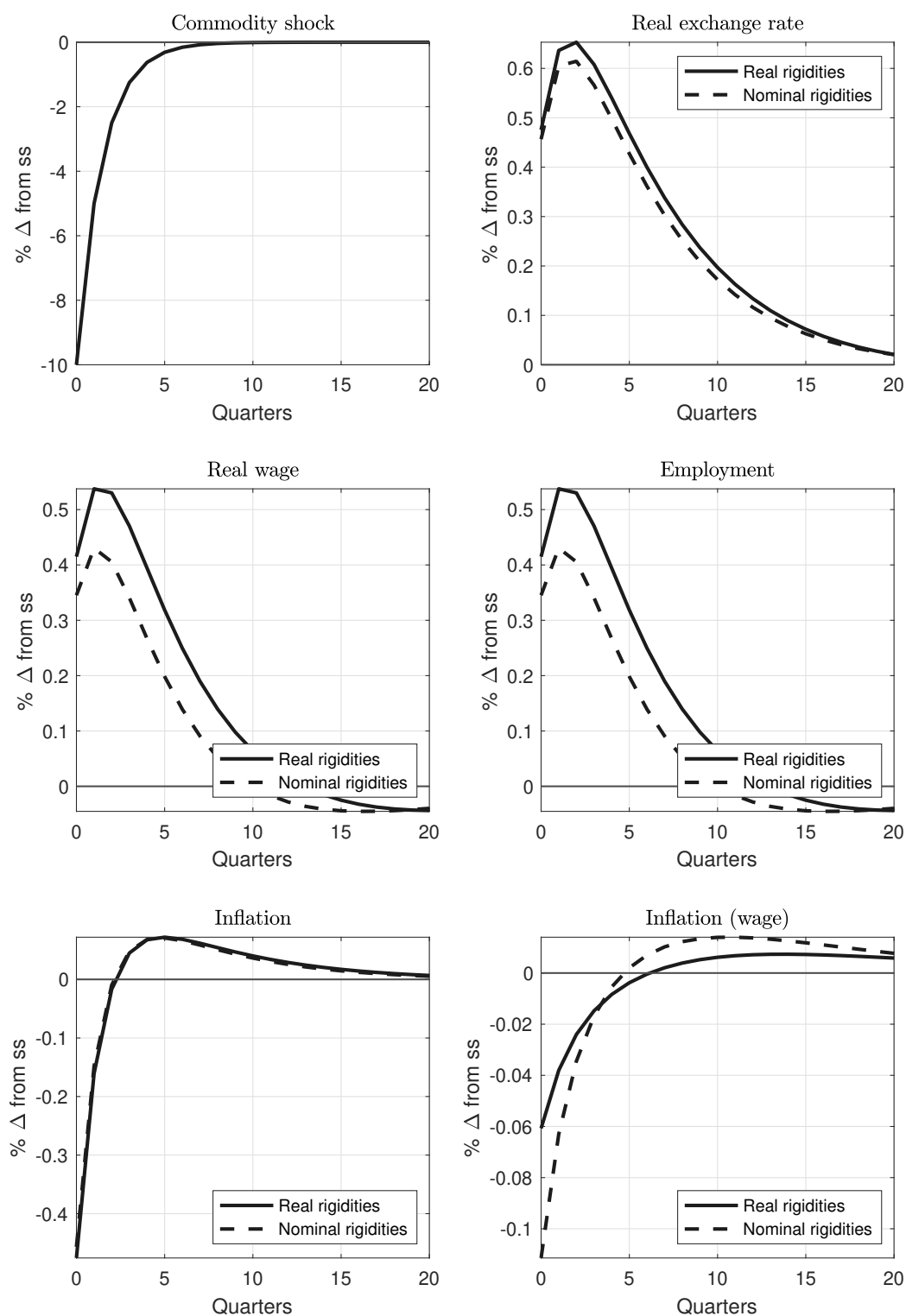
Notes: This figure illustrates the distribution of firm and worker pay components estimated using the AKM procedure. The top figure presents the distribution in 3 dimensions, while the bottom figure shows the corresponding 2-dimensional distribution.

Figure B.10. Impulse response functions to a commodity shock: Taylor rule



Notes: This figure presents impulse response functions to a 10% decline in the dollar-valued endowment of commodities, with the monetary authority following the Taylor rule. The "Real rigidities" lines correspond to the baseline calibration, while the "Nominal rigidities" lines represent the scenario without strategic interactions, where nominal wages are reset every 8 quarters.

Figure B.11. Impulse response functions to a commodity shock: Peg regime



Notes: This figure presents impulse response functions to a 10% decline in the dollar-valued endowment of commodities, with the monetary authority following the Peg regime. The "Real rigidities" lines correspond to the baseline calibration, while the "Nominal rigidities" lines represent the scenario without strategic interactions, where nominal wages are reset every 8 quarters.

C Additional estimations

C.1 Details of AKM estimation

Following Abowd et al. (1999) (AKM), we identify unobservable firm and worker components in their wage outcomes. Our baseline wage specification has the form:

$$\log W_{it} = \lambda_i + \lambda_{J(i,t)} + X_{it}b + \varepsilon_{it},$$

where W_{it} is daily wage in 1,000s of pesos, λ_i is worker i unobservable component; $\lambda_{J(i,t)}$ is firm $J(i, t)$ unobservable component; and vector X_{it} includes unrestricted time fixed effects along with quadratic and cubic functions of age of worker i . Function $J(i, t)$ defines which firm employs worker i at period t .

Table A.1 demonstrates the results from the AKM. Following the literature on the AKM, we perform identification in the largest connected set of firms. However, differently from the AKM literature, the identification of firm pay components is performed in the largest connected set of firms within a metropolitan area. The table shows that we lose less than 1 percent of all firm-worker-year observations when we restrict to the largest connected set. The identified firm and worker components indicate a slightly negative correlation between them.

C.2 Worker transitions

In this subsection, we analyze job-to-job and job-to-“unemployment”-to-job moves. Given that the *PILA* system covers only the formal labor market, workers moving out of the sample do not necessarily enter unemployment. Some of those workers transit to informal labor market. The Colombian formal labor market is represented by workers that earn more relative to their counterparts in the informal labor market. One of the main advantages of the *PILA* system is it provides number of days worked by each employee each month. Therefore, it facilitates our analysis distinguishing between truly job-to-job transitions and transitions that experienced some periods of being out of the sample (being employed in the informal sector or unemployed). As a primary case, we allow workers to be out of the sample between two consecutive jobs for 30 days. This window is in the line with the U.S. Bureau of Labor Statistics that runs the Current Population Survey on a monthly basis.

Figure B.4 plots transition rates defined as the share of workers experiencing job-to-job transitions within 2 weeks, 3 weeks, and 1 month windows. We also plot transition rates with unrestricted duration of “unemployment” spells.

On the other hand, figure B.5 plots transition rates for workers experiencing job-to-job transitions within (i) more than 30 days and less than 90 days, and (ii) more than 90 days. The figure also plots transition rates within and across metropolitan areas. “Within MSA” transition rates represent share of all workers transitioning from job-to-job within the same metropolitan area. “Across MSA” transition rates represent share of workers changing jobs across metropolitan areas. The later might include workers moving from one MSA to another within the same firm.

C.3 Changes in firm identifiers

This paper is centered around strategic interactions between firms in wage setting with competing firms being identified via worker-flows. One common caveat within the literature is that firms could change their administrative identifiers going through a re-structure. Therefore, changes in identifiers could be identified as worker-flows labeling the same firm as two competitors. We follow the procedure developed by Benedetto et al. (2007) using successor-predecessors files. To be specific, we identify a change in firm identifiers if the following conditions meet: (i) 80% co-workers move out of a given firm toward another firm and (ii) firm-successor is an entrant and/or firm-predecessor exits. Once successor and predecessor IDs are identified, we replace the predecessor's identifier with the successor's one.

C.4 General Method of Moments

C.5 Implications to Aggregate Wage Adjustment

$$\mathbb{E} [\omega' \Delta w | \iota, \Delta z] = \frac{\hat{l}_m}{1 - \hat{\lambda}} + \underbrace{\omega' \Delta z \hat{\beta}_1}_{\text{Direct Effect}} + \underbrace{\sum_{s=0}^{\infty} \left[\hat{\lambda}^{s+1} \omega' \mathcal{W}^{s+1} \Delta z \hat{\beta}_1 + \hat{\lambda}^s \omega' \mathcal{W}^s (\mathcal{W} - I) \Delta z \hat{\beta}_2 \right]}_{\text{Indirect Effect}},$$

where ω is a weight vector. Kuersteiner & Prucha (2020) demonstrate that $\hat{\delta} \equiv (\hat{\lambda}, \hat{\beta}_1', \hat{\beta}_2')' \sim \mathcal{N}(\delta_0, \mathbb{V}\mathbb{C}(\hat{\delta}))$. Therefore, using the Delta-method, we can show that direct and indirect effects are distributed normally in the limit and use these properties to derive standard errors for the effects:

$$\begin{aligned} \text{Direct Effect} &\sim \mathcal{N} \left(\omega' \Delta z \beta_{1,0}, \omega' \Delta z \mathbb{V}\mathbb{C}(\hat{\beta}_1) \Delta z' \omega \right), \\ \text{Indirect Effect} &\sim \mathcal{N} \left(\sum_{s=0}^{\infty} [\lambda_0^{s+1} \omega' \mathcal{W}^{s+1} \Delta z \beta_{1,0} + \lambda_0^s \omega' \mathcal{W}^s (\mathcal{W} - I) \Delta z \beta_{2,0}], \right. \\ &\quad \left. \frac{\partial \text{Indirect Effect}}{\partial \hat{\delta}} \mathbb{V}\mathbb{C}(\hat{\delta}) \frac{\partial \text{Indirect Effect}}{\partial \hat{\delta}'} \right), \end{aligned}$$

where $\frac{\partial \text{Indirect Effect}}{\partial \hat{\delta}} \equiv \left(\frac{\partial \text{Indirect Effect}}{\partial \hat{\lambda}}, \frac{\partial \text{Indirect Effect}}{\partial \hat{\beta}_1'}, \frac{\partial \text{Indirect Effect}}{\partial \hat{\beta}_2'} \right)'$ such that

$$\begin{aligned} \frac{\partial \text{Indirect Effect}}{\partial \hat{\lambda}} &= \sum_{s=0}^{\infty} [(s+1) \lambda_0^s \omega' \mathcal{W}^{s+1} \Delta z \beta_{1,0} + s \lambda_0^{s-1} \omega' \mathcal{W}^s (\mathcal{W} - I) \Delta z \beta_{2,0}], \\ \frac{\partial \text{Indirect Effect}}{\partial \hat{\beta}_1} &= \sum_{s=0}^{\infty} \lambda_0^{s+1} \omega' \mathcal{W}^{s+1} \Delta z, \\ \frac{\partial \text{Indirect Effect}}{\partial \hat{\beta}_2} &= \sum_{s=0}^{\infty} \lambda_0^s \omega' \mathcal{W}^s (\mathcal{W} - I) \Delta z. \end{aligned}$$

D Proofs

D.1 Derivation of strategic interactions in wage setting

We use the conjectural variation approach to derive equilibrium wages. To simplify notations, we omit time subscript. Firm j maximizes profit:

$$\begin{aligned} \mathcal{V}_j^f = \max & \left\{ P_j Y_j + \mathcal{E} P_j^* Y_j^* - \sum_{x=1}^{\mathcal{X}} W_{xj} L_{xj} - \mathcal{E} M_j \right\} \\ \text{subject to (3), (6), (7) and } & Y_j + Y_j^* \leq \alpha_j^{-\alpha_j} L_j^{1-\alpha_j} M_j^{\alpha_j}, \\ & L_j = A_j \prod_{x=1}^{\mathcal{X}} L_{xj}^{\beta_{xj}}, \\ & \{H_{xjk}\}, k \neq j \text{ and } j, k \in \mathcal{J}_g. \end{aligned} \quad (\text{D.1.1})$$

where the system of equations $\{H_{xjk}\}$ specifies the market structure. The above problem can be reduced to - once we solve for optimal prices, P_j and P_j^* , and optimal quantity of foreign intermediate inputs, M_j ; and substitute them back into the objective function:

$$\mathcal{V}_j^f = \max_{\{W_{x.}\}_{x=1}^{\mathcal{X}}} \left\{ \mathcal{E}^{\tilde{\gamma}_j - \tilde{\alpha}_j} A_j \prod_{x=1}^{\mathcal{X}} L_{xj}^{\beta_{xj}} - \sum_{x=1}^{\mathcal{X}} W_{xj} L_{xj} \right\} \quad (\text{D.1.2})$$

$$\{H_{xjk}\}, k \neq j \text{ and } j, k \in \mathcal{J}_g.$$

1. We define the set of Lagrangian multipliers as $\lambda_{xjk} W_{xj} L_{xj}$ for each $k \neq j$
2. We define $\epsilon_{xj} = \frac{\partial L_{xj}}{\partial W_{xj}} \frac{W_{xj}}{L_{xj}}$ and $\delta_{xji} = -\frac{\partial L_{xj}}{\partial W_{xi}} \frac{W_{xi}}{L_{xj}}$ as own- and cross-wage labor supply elasticities.
3. To simplify notations, we define marginal product of x -type labor as $\tilde{Y}_{xj} = \beta_{xj} \frac{\partial L_j}{\partial L_{xj}}$

The optimality conditions are:

$$\mathcal{E}^{\tilde{\gamma}_j - \tilde{\alpha}_j} \tilde{Y}_{xj} \frac{\partial L_{xj}}{\partial W_{xj}} - \left(L_{xj} + W_{xj} \frac{\partial L_{xj}}{\partial W_{xj}} \right) + \sum_{k \neq j} \lambda_{xjk} W_{xj} L_{xj} \frac{\partial H_{xjk}}{\partial W_{xj}} = 0, \quad (\text{D.1.3})$$

$$\mathcal{E}^{\tilde{\gamma}_j - \tilde{\alpha}_j} \tilde{Y}_{xj} \frac{\partial L_{xj}}{\partial W_{xi}} - W_{xj} \frac{\partial L_{xj}}{\partial W_{xi}} + \sum_{k \neq j} \lambda_{xjk} W_{xj} L_{xj} \frac{\partial H_{xjk}}{\partial W_{xi}} = 0, \quad \text{for } i \neq j. \quad (\text{D.1.4})$$

After a few manipulations, the above system takes the form:

$$\mathcal{E}^{\tilde{\gamma}_j - \tilde{\alpha}_j} \frac{\tilde{Y}_{xj}}{W_{xj}} \epsilon_{xj} - (1 + \epsilon_{xj}) + \sum_{k \neq j} \lambda_{xjk} W_{xj} \frac{\partial H_{xjk}}{\partial W_{xj}} = 0, \quad (\text{D.1.5})$$

$$-\mathcal{E}^{\tilde{\gamma}_j - \tilde{\alpha}_j} \frac{\tilde{Y}_{xj}}{W_{xj}} \delta_{xji} + \delta_{xji} + \sum_{k \neq j} \lambda_{xjk} W_{xi} \frac{\partial H_{xjk}}{\partial W_{xi}} = 0, \quad \text{for } i \neq j. \quad (\text{D.1.6})$$

By definition, the markdown at firm j for workers of type x is $\mathcal{M}_{xj} \equiv W_{xj}/\mathcal{E}^{\tilde{\gamma}_j - \tilde{\alpha}_j} \tilde{Y}_{xj}$. Using the definition of markdown as well as the own- and cross-wage labor supply elasticities, the optimality conditions can be rewritten as:

$$\mathcal{M}_{xj}^{-1} \epsilon_{xj} - 1 - \epsilon_{xj} + \sum_{k \neq j} \lambda_{xjk} W_{xj} \frac{\partial H_{xjk}}{\partial W_{xj}} = 0, \quad (\text{D.1.7})$$

$$-\mathcal{M}_{xj}^{-1} \delta_{xji} + \delta_{xji} + \sum_{k \neq j} \lambda_{xjk} W_{xi} \frac{\partial H_{xjk}}{\partial W_{xi}} = 0, \quad \text{for } j \neq i. \quad (\text{D.1.8})$$

Let us define λ_{xj} is a vector of all Lagrange multipliers, $\{\lambda_{xji}\}_{i \neq j}$; δ_{xj} is a vector of cross-elasticities, $\{\delta_{xji}\}_{i \neq j}$; h_{xj} is a vector of elements $\left\{W_{xj} \frac{\partial H_{xjk}}{\partial W_{xj}}\right\}_{k \neq j}$ that defines how competitor firms' react to changes in wages of firm j ; \mathcal{H}_{xj} defines elements of matrix $\left\{W_{xi} \frac{\partial H_{xjk}}{\partial W_{xi}}\right\}_{i \neq j, k \neq j}$. Using the notations, the above system takes the following form in the matrix notation:

$$\mathcal{M}_{xj}^{-1} \epsilon_{xj} - 1 - \epsilon_{xj} + h'_{xj} \lambda_{xj} = 0, \quad (\text{D.1.9})$$

$$-\mathcal{M}_{xj}^{-1} \delta_{xj} + \delta_{xj} + \mathcal{H}_{xj} \lambda_{xj} = 0. \quad (\text{D.1.10})$$

Solving for λ_{xj} from equation (D.1.10) and substituting it to equation (D.1.9) gives us the equilibrium markdown as a function of the own- and cross-wage labor supply elasticities as well as the system $\{H_{xjk}\}$ that defines market structure.

Defining the markdown through the perceived labor supply elasticity, σ_{xj} , as $\mathcal{M}_{xj} = \frac{\sigma_{xj}}{\sigma_{xj} + 1}$, one can solve for the perceived labor supply elasticity:

$$\sigma_{xj} = \epsilon_{xj} + h'_{xj} \mathcal{H}_{xj}^{-1} \delta_{xj} \equiv \epsilon_{xj} - \sum_{k \neq j} \delta_{xjk} \times \kappa_{xjk}, \quad (\text{D.1.11})$$

where κ_{xj} is a vector that solves the following system $-\mathcal{H}'_{xj} \kappa_{xj} = h_{xj}$. κ_{xj} is a vector of wages' changes of all firms except firm i in response to change a one percent change in firm j 's wage that sustain the Nash equilibrium. In other words, $\kappa_{xj} = \{\kappa_{xjk}\}_{k \neq j} = \left\{\frac{\Delta W_{xk}/W_{xk}}{\Delta W_{xj}/W_{xj}}\right\}_{k \neq j}$ for each $x = 1, \dots, \mathcal{X}$.

Given that the equilibrium markdown depend on W_{xj} and $\{W_{xk}\}_{k \neq j}$, one can write the equilibrium wage relationship as:

$$\log W_{xj} = \log \tilde{Y}_{xj} + (\tilde{\gamma}_j - \tilde{\alpha}_j) \log \mathcal{E} + \log \mathcal{M}_{xj} \left(W_{xj}, \{W_{xk}\}_{k \neq j} \right). \quad (\text{D.1.12})$$

Next, we specify the system of equations $\{H_{xjk}\}$ that specify various market structures and solve for a closed form solution of the markdown. To be specific, we consider three main workhorse market structures considered in the macro-IO literature such as **Cournot-Nash** competition, **Bertrand-Nash** competition, and **Collusion**.

Under the **Cournot-Nash** competition, system H_{-xj} is a vector of $\{H_{xjk}\}_{k \neq j}$, and for j and k are

in group $g \in 1, \dots, \mathcal{G}$ with H_{xjk} being:

$$H_{xjk} = \frac{\left(W_{xkt}^{\phi_x} V_{xk} Z_{kt}^{\beta_z}\right)^{\frac{1}{1-\rho_x}}}{D_{xg}} \frac{D_{xg}^{1-\rho_x}}{\sum_{g'=0,1,\dots,\mathcal{G}} D_{xg'}^{1-\rho_x}} - L_{xk}^*, \quad k \neq j \text{ and } k \in \mathcal{J}_g, \quad (\text{D.1.13})$$

where $D_{xg} = \sum_{k' \in \mathcal{J}_g} \left(W_{xk't}^{\phi_x} V_{xk'} Z_{k't}^{\beta_z}\right)^{\frac{1}{1-\rho_x}}$. Given that one can compute $W_{xj} \frac{\partial H_{xjk}}{\partial W_{xj}}$ for each $k \neq j$ and $W_{xi} \frac{\partial H_{xjk}}{\partial W_{xi}}$ for each $k, i \neq j$ such that:

$$W_{xi} \frac{\partial H_{xjk}}{\partial W_{xi}} = \begin{cases} \frac{\phi_x}{1-\rho_x} L_{xk} (1 - \rho_x L_{k|xg} - (1 - \rho_x) L_{xk}) & \text{if } i = k, \\ -\frac{\phi_x}{1-\rho_x} L_{xk} (\rho_x L_{i|xg} + (1 - \rho_x) L_{xi}) & \text{if } i \neq k, \end{cases} \quad (\text{D.1.14})$$

where $L_{xk} = L_{k|xg} L_{xg}$, $L_{k|xg} = \left(W_{xkt}^{\phi_x} V_{xk} Z_{kt}^{\beta_z}\right)^{\frac{1}{1-\rho_x}} / D_{xg}$, and $L_{xg} = D_{xg}^{1-\rho_x} / \sum_{g' \in 0,1,\dots,\mathcal{G}} D_{xg'}^{1-\rho_x}$. Using equations (D.1.14) to generate \mathcal{H}_{xj} and h_{xj} , one can solve for $\kappa_{xj} = \{\kappa_{xjk}\}_{k \neq j}$ such that:

$$\kappa_{xjk} = \frac{L_{j|xg} (\rho_x + (1 - \rho_x) L_{xg})}{1 - (1 - L_{j|xg}) (\rho_x + (1 - \rho_x) L_{xg})}. \quad (\text{D.1.15})$$

Combining the equilibrium κ_{xjk} with the own labor supply elasticity, $\epsilon_{xj} = \frac{\phi_x}{1-\rho_x} (1 - \rho_x L_{j|xg} - (1 - \rho_x) L_{xj})$, and the cross labor supply elasticity, $\delta_{xjk} = \frac{\phi_x}{1-\rho_x} (\rho_x L_{k|xg} + (1 - \rho_x) L_{xk})$, we can derive the closed form solution for perceived labor supply elasticity of firm j of x -type workers:

$$\sigma_{xj} = \frac{\phi_x}{1 - \rho_x} \left(1 - \frac{L_{j|xg} (\rho_x + (1 - \rho_x) L_{xg})}{1 - (1 - L_{j|xg}) (\rho_x + (1 - \rho_x) L_{xg})} \right). \quad (\text{D.1.16})$$

$$\frac{\partial \mathcal{M}_{xj}}{\partial w_{xk}} = \frac{\tilde{\sigma}_{xj} + 1}{\tilde{\sigma}_{xj}} \begin{cases} -\frac{\phi_x^2 \rho_x}{1-\rho_x} \frac{N_{j|g}(1-N_g)(\rho(1-N_{j|g})+(1-\rho)N_g)}{(1-(1-N_{j|g})(\rho_x+(1-\rho_x)N_g))^2} & \text{if } k = j, \\ \frac{\phi_x^2 \rho_x}{1-\rho_x} \frac{N_{j|g} N_{k|g} (1-N_g)}{(1-(1-N_{j|g})(\rho_x+(1-\rho_x)N_g))^2} & \text{if } k \neq j, \\ \frac{\phi_x^2}{1-\rho_x} \frac{N_{j|g} N_{k|g} N_g N_{g'}}{(1-(1-N_{j|g})(\rho_x+(1-\rho_x)N_g))^2} & \text{if } k \neq j \text{ and } k \in \mathcal{J}_{g'} \end{cases} \quad (\text{D.1.17})$$

Under **Bertrand-Nash** competition, system H_{-xj} is a vector of $\{H_{xjk}\}_{k \neq j}$, and for j and k are in group $g \in 1, \dots, \mathcal{G}$ with H_{xjk} being:

$$H_{xjk} = W_{xk} - W_{xk}^*, \quad k \neq j \text{ and } k \in \mathcal{J}_g. \quad (\text{D.1.18})$$

Alternatively to the Cournot assumption, the Bertrand assumption means that firms commit to their optimal wages rather than quantities of labor:

$$W_{xi} \frac{\partial H_{xjk}}{\partial W_{xi}} = \begin{cases} W_{xi} & \text{if } i = k, \\ 0 & \text{if } i \neq k, \end{cases} \quad (\text{D.1.19})$$

Equations (D.1.19) lead to zero equilibrium response in wages, $\kappa_{xjk} = 0$. This result is in the line with the Bertrand assumption meaning that firm j does not “perceive” reaction in wages by their competitors. Therefore, the perceived labor supply elasticity of firm j for x -type workers is:

$$\sigma_{xj} = \frac{\phi_x}{1 - \rho_x} (1 - \rho_x L_{j|xg} - (1 - \rho_x) L_{xj}). \quad (\text{D.1.20})$$

$$\frac{\partial \mathcal{M}_{xj}}{\partial w_{xk}} = \frac{\tilde{\sigma}_{xj} + 1}{\tilde{\sigma}_{xj}} \begin{cases} -\frac{\phi_x^2}{(1-\rho_x)^2} ((1 - N_{j|g}) (\rho + (1 - \rho) N_g) + (1 - \rho_x)^2 N_g (1 - N_g) N_{j|g}) & \text{if } k = j, \\ \frac{\phi_x^2}{(1-\rho_x)^2} N_{j|g} N_{k|g} (\rho + \rho (1 - \rho) N_g + (1 - \rho)^2 N_g^2) & \text{if } k \in \mathcal{J}_g, \\ \phi_x^2 N_{j|g} N_{k|g'} N_g N_{g'} & \text{if } k \in \mathcal{J}_{g'}, \end{cases} \quad (\text{D.1.21})$$

Under **collusion**, all firms in group \mathcal{J}_g commit to same wage paid to workers of type x ; therefore, system $H_{-xj} = \{H_{xjk}\}_{k \neq j}$ can be formulated such that each element H_{xjk} is:

$$H_{xjk} = W_{xk} - W_{xj}, \quad k \neq j \text{ and } k, j \in \mathcal{J}_g. \quad (\text{D.1.22})$$

$$W_{xi} \frac{\partial H_{xjk}}{\partial W_{xi}} = \begin{cases} W_{xk} & \text{if } i = k, \\ -W_{xj} & \text{if } i = j, \\ 0 & \text{if } i \neq j, k, \end{cases} \quad (\text{D.1.23})$$

Opposite to the **Bertrand**-Nash competition, the collusion delivers uniform change in wages in a given group, $\kappa_{xjk} = 1$, meaning that the perceived labor supply elasticity is:

$$\sigma_{xj} = \phi_x (1 - L_{xg}). \quad (\text{D.1.24})$$

$$\frac{\partial \mathcal{M}_{xj}}{\partial w_{xk}} = \frac{\tilde{\sigma}_{xj} + 1}{\tilde{\sigma}_{xj}} \begin{cases} -\phi_x^2 N_g (1 - N_g) N_{k|g} & \text{if } k = j \text{ or } k \in \mathcal{J}_g, \\ \phi_x^2 N_g N_{g'} N_{k|g'} & \text{if } k \in \mathcal{J}_{g'}, \end{cases} \quad (\text{D.1.25})$$

Alternatively, one can consider a case, where $\kappa_{xjk} = \kappa_{xj}$ for any $k \neq j$ such that $\kappa_{xj} \in [0, 1)$.

D.2 General equilibrium model

The general equilibrium consists of five blocks: final and intermediate good producers, workers, households, and the rest of world block, and it is described by 21 equations. Given parameters $\{\mu_j, \mu_j^*, \alpha_j\}_{j=1, \dots, \mathcal{J}}$, we are able to solve for $\{Y_{jt}, Y_{jt}^*, P_{jt}, P_{jt}^*, L_{jt}, M_{jt}, \psi_{jt}\}_{j=1, \dots, \mathcal{J}}$.

Final good producers. Final good producers take prices $\{P_{jt}, P_{jt}^*\}_{j=1, \dots, \mathcal{J}}$ and shares $\{\mu_j, \mu_j^*\}_{j=1, \dots, \mathcal{J}}$ as given; and they maximize their profits choosing $\{Y_{jt}, Y_{jt}^*\}_{j=1, \dots, \mathcal{J}}$. The optimality conditions of the final good producers' maximization problem lead to the demand for products of firm j such that:

$$Y_{jt} = \mu_j P_{jt}^{-\eta} \mathcal{Y}_t \quad (\text{D.2.1})$$

$$Y_{jt}^* = \mu_j^* P_{jt}^{*-\eta} \mathcal{Y}_t^*, \quad (\text{D.2.2})$$

where \mathcal{Y}_t and \mathcal{Y}_t^* represent amount of final goods produced for domestic and foreign markets, respectively:

$$\mathcal{Y}_t = \left(\sum_{j=1}^{\mathcal{J}} \mu_j^{\frac{1}{\eta}} Y_{jt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad (\text{D.2.3})$$

$$\mathcal{Y}_t^* = \left(\sum_{j=1}^{\mathcal{J}} \mu_j^{*\frac{1}{\eta}} Y_{jt}^{*\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \quad (\text{D.2.4})$$

Given the normalization of prices of final goods to one, prices of intermediate good producers should satisfy the following conditions:

$$\sum_{j=1}^{\mathcal{J}} \mu_j P_{jt}^{1-\eta} = 1, \quad (\text{D.2.5})$$

$$\sum_{j=1}^{\mathcal{J}} \mu_j^* P_{jt}^{*1-\eta} = 1. \quad (\text{D.2.6})$$

Intermediate good producers. Each intermediate good producer j , $j = 1, \dots, \mathcal{J}$, takes exchange rate \mathcal{E}_t and parameters $\{\alpha_j, \mu_j, \mu_j^*\}_{j=1, \dots, \mathcal{J}}$ as given. They maximize their profit by choosing $\{P_{jt}, P_{jt}^*, M_{jt}\}_{j=1, \dots, \mathcal{J}}$ and $\{W_{xjt}\}_{x=1, \dots, \mathcal{X}; j=1, \dots, \mathcal{J}}$ with the Lagrangian multiplier for the capacity constraint $\{\psi_{jt}\}_{j=1, \dots, \mathcal{J}}$. Optimal prices of intermediate good producer j are defined according to

$$P_{jt} = \frac{\eta}{\eta-1} \psi_{jt}, \quad (\text{D.2.7})$$

$$\mathcal{E}_t P_{jt}^* = \frac{\eta}{\eta-1} \psi_{jt}. \quad (\text{D.2.8})$$

The optimal quantity of foreign intermediate goods used in production is defined by:

$$M_{jt} = \left(\frac{\psi_{jt}}{\mathcal{E}_t} \right)^{1/1-\alpha_j} \alpha_j L_{jt}. \quad (\text{D.2.9})$$

Therefore, given that $F_j(L_{jt}, M_{jt}) = \alpha_j^{-\alpha_j} L_{jt}^{1-\alpha_j} M_{jt}^{\alpha_j}$, the marginal effect of labor on revenue is $\psi_{jt} \left(\frac{\psi_{jt}}{\mathcal{E}_t} \right)^{\alpha_j/1-\alpha_j}$. To solve for ψ_{jt} , we use the capacity constraint in the equilibrium:

$$Y_{jt} + Y_{jt}^* = F_j(L_{jt}, M_{jt}), \quad (\text{D.2.10})$$

where equilibrium demand ((D.2.1) and (D.2.2)) is determined at equilibrium prices. We can show that $\frac{d\psi_{jt}}{\psi_{jt}} \approx \frac{\gamma_{jt}\eta + \frac{\alpha_j}{1-\alpha_j}}{\eta + \frac{\alpha_j}{1-\alpha_j}} \frac{d\mathcal{E}_t}{\mathcal{E}_t}$ in the equilibrium. Therefore, the marginal effect of labor on profit $\psi_{jt} \left(\frac{\psi_{jt}}{\mathcal{E}_t} \right)^{\alpha_j/1-\alpha_j} \approx \mathcal{E}_t^{\frac{\gamma_{jt}\eta - (\eta-1)\alpha_j}{\eta - (\eta-1)\alpha_j}} = \mathcal{E}_t^{\tilde{\gamma}_j - \tilde{\alpha}_j}$, where $\tilde{\gamma}_j = \frac{\gamma_{jt}\eta}{\eta - (\eta-1)\alpha_j}$, $\tilde{\alpha}_j = \frac{(\eta-1)\alpha_j}{\eta - (\eta-1)\alpha_j}$, and $\gamma_{jt} = \frac{\mu_j^* \mathcal{Y}_t^* \mathcal{E}_t}{\mu_j \mathcal{Y}_t + \mu_j^* \mathcal{Y}_t^* \mathcal{E}_t}$.

Wages are settled according to:

$$W_{xjt} = \psi_{jt} \left(\frac{\psi_{jt}}{\mathcal{E}_t} \right)^{\alpha_j/1-\alpha_j} \beta_x \frac{L_{jt}}{L_{xjt}} \times \mathcal{M}_{xjt}, \quad (\text{D.2.11})$$

where $\mathcal{M}_{xjt} = \frac{\sigma_{xjt}}{\sigma_{xjt} + 1}$ and σ_{xjt} is perceived labor supply elasticity. The exact form of the labor supply elasticity depends on the market structure:

$$\sigma_{xjt} = \begin{cases} \frac{\phi_x}{1-\rho_x} \left(1 - \rho_x L_{j|xg} - (1-\rho_x) L_{xj} \right), & \text{under Bertrand-Nash,} \\ \frac{\phi_x}{1-\rho_x} \left(1 - \frac{L_{j|xg}(\rho_x + (1-\rho_x)L_{xg})}{1 - (1-L_{j|xg})(\rho_x + (1-\rho_x)L_{xg})} \right), & \text{under Cournot-Nash} \\ \phi_x (1 - L_{xg}). & \text{under Collusion} \end{cases}$$

Workers. Workers of type x take wages and unobserved firm amenities (time-invariant and time-variant) as given $\{W_{xjt}, V_{xj}, Z_{jt}\}_{j=1, \dots, \mathcal{J}}$ and shares $\{\mu_j, \mu_j^*\}_{j=1, \dots, \mathcal{J}}$. This results in the labor supply (it coincides with equilibrium allocation of labor across firms) that is defined according to:

$$L_{xjt}(W_{xjt}; \mathbf{W}_{-xjt}) = \frac{\left(W_{xjt}^{\phi_x} V_{xj} Z_{jt}^{\beta_z} \right)^{\frac{1}{1-\rho_x}}}{D_g} \frac{D_g^{1-\rho_x}}{\sum_{g' \in \{0, 1, \dots, G\}} D_{g'}^{1-\rho_x}}. \quad (\text{D.2.12})$$

Households. Households take price \mathcal{P}_t , exchange rate \mathcal{E}_t , interest rate on net foreign assets i_t^* , profit of firms $\{\mathcal{V}_{jt}^f\}_{j=1, \dots, \mathcal{J}}$, oil endowment \mathcal{O}_t , and wages earned $\{W_{xjt} L_{xjt}\}_{x=1, \dots, \mathcal{X}; j=1, \dots, \mathcal{J}}$, accumulated net foreign assets \mathcal{B}_{t-1}^* , and they decide on total consumption \mathcal{C}_t , consumption of domestic and foreign goods, \mathcal{C}_{Ht} and \mathcal{C}_{Ft} , and position in net foreign assets \mathcal{B}_t^* .

Equation D.2.13 characterize optimal households' accumulation of net foreign savings:

$$1 = (1 + i_t^*) \beta \mathbb{E}_t \left\{ \frac{\mathcal{C}_t}{\mathcal{C}_{t+1}} \frac{\mathcal{P}_t}{\mathcal{P}_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\}, \quad (\text{D.2.13})$$

Equations (D.2.14) to (D.2.16) characterize households' demand for domestic and foreign goods

as well as the consumption bundle aggregator

$$\mathcal{C}_{Ht} = \omega \mathcal{P}_t \mathcal{C}_t, \quad (\text{D.2.14})$$

$$\mathcal{E}_t \mathcal{C}_{Ft} = (1 - \omega) \mathcal{P}_t \mathcal{C}_t, \quad (\text{D.2.15})$$

$$\mathcal{C}_t = \left(\frac{\mathcal{C}_{Ht}}{\omega} \right)^\omega \left(\frac{\mathcal{C}_{Ft}}{1 - \omega} \right)^{1 - \omega}. \quad (\text{D.2.16})$$

The definition of the consumption bundle leads to the definition of aggregate price level is defined according to $\mathcal{P}_t = \mathcal{E}^{1 - \omega}$.

The Rest of the World. Equations (D.2.17) to (D.2.18) close the general equilibrium model by defining external shocks in terms of commodity price shocks and shocks to capital flows:

$$\log \mathcal{O}_t - \log \bar{\mathcal{O}} = \rho_o (\log \mathcal{O}_{t-1} - \log \bar{\mathcal{O}}) + \epsilon_{ot}, \quad (\text{D.2.17})$$

$$i_t^* = \bar{i}^* + \psi \frac{\mathcal{B}_t^* - \bar{\mathcal{B}}^*}{\bar{\mathcal{B}}^*} + \epsilon_{i^*t}, \quad (\text{D.2.18})$$

$$\epsilon_{i^*t} = \rho_{i^*} \epsilon_{i^*t-1} + \varepsilon_t. \quad (\text{D.2.19})$$

Market clearing. Equation (D.2.20) defines market clearing in domestic final good's market. Equation (D.2.21) is a combination of the household budget constraint, firms' profit, resource constraint and defines balance of payments:

$$\mathcal{C}_{Ht} = \mathcal{Y}_t, \quad (\text{D.2.20})$$

$$\mathcal{B}_t^* = \mathcal{O}_t + \mathcal{Y}_t^* - \mathcal{C}_{Ft} - \mathcal{M}_t + (1 + i_{t-1}^*) \mathcal{B}_{t-1}^*. \quad (\text{D.2.21})$$